Radicals

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Class Discussion

Simplify $\sqrt{3+2\sqrt{2}}$.

MAML 2010. Simplify $\sqrt{5/4-\sqrt{3/2}}-\sqrt{5/4+\sqrt{3/2}}$.

Solve $\sqrt[3]{13x+37}=\sqrt[3]{2}+\sqrt[3]{13x-37}$.

Warm-Up

Exercise 1. You're in the basement of a house that has three light switches, all in the off position. You know that one of the switches controls a lightbulb in the study upstairs and you want to find out which one it is. You can use the switches as many times a you want but you are allowed to go upstairs only once. How can you do it?

Exercise 2. You have a basket containing five oranges. You have five hungry friends. You give each of your friends one orange. After the distribution each of your friends has one orange each, yet there is an orange remaining in the basket. How can it be?

Exercise 3. A man sees a boat that is full of people. And yet there isn't a single person on the boat. How is this possible?

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Exercise 4. Simplify $\sqrt{49+28\sqrt{3}}$.

Exercise 5. AHSME. Solve the equation $\sqrt{5x-1}+\sqrt{x-1}=2$.

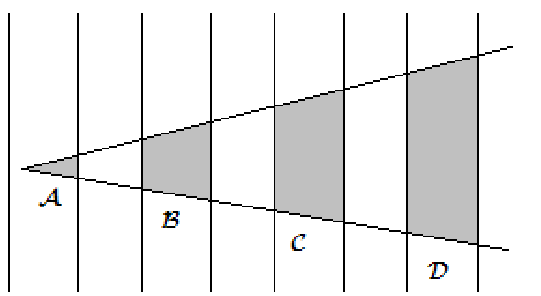
Exercise 6. ARML. Find the ordered pair of positive integers $(a,\ b)$, with $a < b$, for which $\sqrt{1+\sqrt{21+12\sqrt{3}}} = \sqrt{a}+\sqrt{b}$.

Exercise 7. AIME. Find the value of $(52+6\sqrt{43})^{3/2} - (52-6\sqrt{43})^{3/2}$.

Competition Practice

Exercise 8. 2006 AIME-I. Let $\mathcal{S}$ be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$ where $a,\ b,\ c$ are distinct digits. Find the sum of the elements of $\mathcal{S}$.

Exercise 9. 2006 AIME-I. An angle is drawn on a set of equally spaced parallel lines as shown. The ratio of the area of shaded region $\mathcal{C}$ to the area of shaded region $\mathcal{B}$ is 11/5. Find the ratio of shaded region $\mathcal{D}$ to the area of shaded region $\mathcal{A}$.



Exercise 10. 2006 AIME-I. Hexagon $ABCDEF$ is divided into five rhombuses, $\mathcal{P},\ \mathcal{Q},\ \mathcal{R},\ \mathcal{S}$, and $\mathcal{T}$, as shown. Rhombuses $\mathcal{P},\ \mathcal{Q},\ \mathcal{R}$ and $\mathcal{S}$ are congruent, and each has area $\sqrt{2006}$. Let K be the area of rhombus $\mathcal{T}$. Given that K is a positive integer, find the number of possible values for K.