

Pythagorean Theorem

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Einstein-Pythagorean equation $E = m(a^2 + b^2)$.

Class Discussion

Different proofs of the Pythagorean theorem. For coprime integers m and n of different parity, $n^2 - m^2$, $2mn$, $m^2 + n^2$ form a primitive Pythagorean triple. A proof that a primitive Pythagorean triangle has an odd and an even leg.

Warm-up

Exercise 1. Show how you can plant 7 trees and end up with 6 rows of 3 trees each.

Exercise 2. Nine dots are arranged into a three by three grid. What is the smallest number of squares needed to separate each of the dots?

Exercise 3. Find all rectangles with integer sides so that their perimeter equals their area.

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Exercise 4. One of the legs of a right triangle is 10 meters longer than the other leg, and 10 meters shorter than the hypotenuse. Find the hypotenuse.

Exercise 5. Prove that in a right trapezoid, the difference of squares of the diagonals equals the difference of squares of the parallel sides.

Exercise 6. Find all Pythagorean triangles such that one leg is shorter than the hypotenuse by 1.

Exercise 7. Two vertices of a square lie on a circle of radius R , the other two — on a tangent to the circle. Find the diagonal of the square.

Exercise 8. The hypotenuse of a right triangle is a side of a square that lies on the other side of the triangle. Find the distance from the triangle vertex that corresponds to the right angle to the center of the square. The legs of the triangle are a and b .

Competition Practice

Exercise 9. 2003 AMC 10A. Simplify $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}$.

Exercise 10. 2003 AMC 10A. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

Exercise 11. 2003 AMC 10A. A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

Exercise 12. 2005 AMC 10A. In $\triangle ABC$ we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on \overline{AB} and \overline{AC} respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

Exercise 13. Linguistics Olympiad. The following list shows some numbers, written by words in the language of some Pacific island nation. Each next number is equal to the previous one plus 2. Can you determine what these numbers are?

- thabung ke nua lo
- thabung ke nua vak
- libenyita ke nua khasa
- libenyita ke nua kun
- libenyita ke nua thabung
- libenyita ke nua thabung ke nua lo
- libenyita ke nua thabung ke nua vak