Euclid's Algorithm

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October 26, 2015

Nothing produces such odd results as trying to get even.

Class Discussion

Euclid's algorithm. Extended Euclid's algorithm.

Warm Up

Exercise 1. 2002 AMC 8. Harold tosses a nickel four times. What is the probability that he gets at least as many heads as tails?

Exercise 2. 2002 AMC 8. In a mathematics contest with ten problems, a student gains 5 points for a correct answer and loses 2 points for an incorrect answer. If Olivia answered every problem and her score was 29, how many correct answers did she have?

Exercise 3. 2002 AMC 8. How many whole numbers between 99 and 999 contain exactly one 0?

Euclid's Algorithm

Exercise 4. Prove that $\gcd(a+b, \operatorname{lcm}(a,b)) = \gcd(a,b)$.

Exercise 5. Find five positive integers such that the difference of any two of them is equal to the greatest common divisor of that two numbers.

Exercise 6. Prove that numbers $27x+4$ and $18x+3$ are coprime for any integer x.

Exercise 7. Can the greatest common divisor of two numbers be bigger than their difference?

Exercise 8. Prove that the product of any two consecutive positive integers can't be a non-trivial power of an integer.

Exercise 9. Let a and b be positive integers such that $56a = 65b$. Prove that $a + b$ is composite.

Exercise 10. Prove that $\gcd(5a + 3b,\ 13a + 8b) = \gcd(a, b)$.

Exercise 11. 11 girls and n boys went to a park to collect pine cones for their Fibonacci project. They collected $n^2 + 9n - 2$ pine cones together. Given that each person collected the same number of cones, can you tell if there were more boys or girls in the group?

Exercise 12. Find the greatest common divisor of two consecutive Fibonacci numbers.

Exercise 13. Find the greatest common divisor of two numbers $111\ldots111$ and $111\ldots111$. The first number is written with 100 ones and the second with 60 ones.

Exercise 14. Find all natural numbers $n > 1$, for which $n^3 - 3$ is divisible by $n - 1$.

Exercise 15. A container contains 12 litres of water. You have two more containers for 5 and 7 litres each. Divide water into two halves.

Challenge Problems

Exercise 16. 2006 Qualifying quiz for Canada/USA Mathcamp. Seven men are sitting in a room. Someone puts a hat on the head of each man. Each hat has an equal probability of being one of the seven colors of the rainbow. It is okay for two men to have hats of the same color. Without communicating with each other, each man guesses the color of the hat on his head. If at least one of them guesses right, they win this little game of theirs. If they are allowed to create a strategy beforehand, how can they be assured of winning?