# Chinese Remainder Theorem

Tanya Khovanova

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## Class Discussion

Hat problem. Chinese remainder theorem.

## Warm Up

**Exercise 1.** What letter of the alphabet is the one which appears in this sentence eight letters before the letter which appears five letters after the fourth appearance of the first letter to occur four times in this sentence?

**Exercise 2.** Three days ago, yesterday was the day before Sunday. What day will it be tomorrow?

**Exercise 3.** What occurs once in every minute, twice in every moment, but never in a thousand years?

**Exercise 4.** Two boys wish to cross a river. The only way to get to the other side is by boat, but that boat can only take one boy at a time. The boat cannot return on its own, there are no ropes or similar tricks, yet both boys manage to cross using the boat. How?

**Exercise 5.** What is the answer to this question?

**Exercise 6. 1964 MAML.** The tens digit of a certain three-digit number is equal to the sum of the hundreds digits and units digit. The number is always divisible by ...?

**Exercise 7. 1964 MAML.** If the geometric mean between two numbers, $p$ and $q$, is 3 and the sum of their squares is 8, then what is $(p-q)^2$?

**Exercise 8. 1964 MAML.** The difference of the squares of two odd numbers is always divisible by a) 3, b) 5, c) 6, d) 7, e) 8?

## Chinese Remainder Theorem

**Exercise 9.** What can you say if the number of divisors of a number including 1 and itself is 2? 3?

**Exercise 10.** The product of two numbers is 1000. Each of the numbers is not divisible by 10. Find their sum.

**Exercise 11.** Peter prepared some lollipops for his party. If he divides them evenly for five people, then two pieces will be left over. If he divides them for four people one lollipop will be left. But he can divide them evenly for three people. What the smallest number of lollipops Peter could have had?

**Exercise 12.** Find the smallest natural number that have remainders 1, 2, 4, 6 when divided by 2, 3, 5, 7 correspondingly.

**Exercise 13.** Find remainders:

1. $19^{10}$ modulo 66
2. $19^{14}$ modulo 70
3. $17^9$ modulo 48
4. $14^{14^{14}}$ modulo 100

**Exercise 14.** Replace stars with digits so that 454** is divisible by 2, 7 and 9.

## Competition Practice

**Exercise 15. 1964 MAML.** If the discriminant of $px^2 + 2qx + r = 0$, then

1. $p$, $q$ and $r$ form an arithmetic progression
2. $p$, $q$ and $r$ are equal
3. $p$, $q$ and $r$ form a geometric progression
4. $p$, $q$ and $r$ are all negative
5. none of the above.

**Exercise 16. 2009 HMNT.** A *computer program* is a function that takes in 4 bits, where each bit is either a 0 or a 1, and outputs TRUE or FALSE. How many computer programs are there?

**Exercise 17.** Find all possible integers $N$ that are products of distinct primes, such that $N$ is divisible by every prime divisor reduced by one.