

# Parity

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Two bytes walk into a bar. The first byte turns to the second and says “I think I may have a parity error.” The second byte turns to the first and says “yeah, you look a bit off.”

## Class Discussion

Parity arguments. 100 members of the math club are voting whether to chose Mike or Kate as there president. Everyone voted. Kate had 23 more votes than Mike. She immediatly requested a recount. Why? The ten-card trick.

## Warm Up

**Exercise 1.** In a 5-story building, 1 person lives on the first floor, a couple lives on the second floor, three people live on the third floor and four people live on the fourth floor. The building has an elevator. On what floor it stops the most often?

**Exercise 2.** Prove that the square of an odd number has remainder 1 modulo 8.

## Parity. Remainders

**Exercise 3.** Tanya bought a notebook with 88 pages. She numbered all the pages from 1 to 176. Mike snatched 25 pages from the notebook and summed up the page numbers. He got 2014. Tanya immediatly told him that he was wrong? How did she know?

**Exercise 4.** Find all pairs of prime numbers that have a prime difference and a prime sum.

**Exercise 5.** Prove that two consecutive Fibonacci numbers are coprime.

**Exercise 6.** 25 lazy students were standing on a straight line all facing the same direction that is perpendicular to the line. The Math Teacher said: “Everybody turn to the left” after which some students turned to the left, some turned to the right, and some didn’t move. Is it always true that the Math Teacher can now insert herself into the line in such a way that the numbers of students facing her on both of her sides will be the same?

**Exercise 7.** Can you make a magic square out of the first 9 prime numbers?

**Exercise 8.** I have counted my candy and noticed that if I divide the candy into two equal piles, then one piece of candy will be left over. If I divide the candy into three equal piles, then 2 pieces of candy will be left over, if I divide it into 4 piles, then 3 pieces of candy will be left over. If I divide it into five piles, then 4 pieces of candy will be left over, and if I divide the candy into six piles, then 5 pieces of candy will be left over. How much candy do I have?

## Competition practice

**Exercise 9. 1986 AIME.** What is the largest positive integer  $n$  for which  $n^3 + 100$  is divisible by  $n + 10$ ?

**Exercise 10. 1984 AIME.** The integer  $n$  is the smallest positive multiple of 15 such that every digit of  $n$  is either 8 or 0. Compute  $n/15$ .

**Exercise 11. 1983 AIME.** Let  $a_n$  equal  $6^n + 8^n$ . Determine the remainder upon dividing  $a_{83}$  by 49.

## Challenge Problems

**Exercise 12.** The squares on the 5 by 5 checker-board are numbered 1 through 25. There are 25 checkers that are also numbered 1 through 25. Can you put all the checkers on the checker-board (one checker per square), so that each checker is on the square adjacent (horizontally or vertically) to its number?