

Algebraic Expressions

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Class Discussion

Factor: $x^2 - y^2$, $x^3 - y^3$, $x^3 + y^3$, $x^n - y^n$, $x^n + y^n$.

Expand: $(x + y)^2$, $(x + y)^3$, $(x + y)^4$, $(x + y)^n$, $(x - y)^n$. Pascal's triangle.

Factoring polynomials with real coefficients.

Warm Up

Exercise 1. Genetically engineered octopuses with an even number of arms always tell the truth, the ones with an odd number of arms always lie. Only octopuses with six, seven or eight arms are allowed to serve Neptune. The four guards from the night shift at Neptune's palace were bored, and they started to argue: The magenta one said, "All together we have 31 arms." The cyan one said, "No, we do not." The brown one said, "The beige one has six arms." The beige one said, "You, brown, are lying."

Who is lying and who is telling the truth?

Exercise 2. Solve the equation $x(x + 1) = 2014 \cdot 2015$.

Exercise 3. Find the sum of all the coefficients of $(x^2 - 3x + 1)^{100}$.

Algebraic Expressions

Exercise 4. Factor: $x^6 - y^6$ and $x^8 - y^8$.

Exercise 5. Expand: $(x - y)^6$.

Exercise 6. Simplify:

$$\frac{\left(\sqrt{a^2 + a\sqrt{a^2 - b^2}} - \sqrt{a^2 - a\sqrt{a^2 - b^2}}\right)^2}{2\sqrt{a^2b}} \div \left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} - 2\right).$$

Exercise 7. 1990 MAΘ. Factor completely:

$$-a^2b^2 + 2ab^3 - b^4 + a^2c^2 - 2abc^2 + b^2c^2.$$

Exercise 8. 1991 MAΘ. Simplify the following expression as much as possible:

$$\left(\frac{a^3 - 1}{a^2 - 1}\right) \left(\frac{a^2 + 2a + 1}{a^3 + 1}\right) \left(\frac{a^2 - a + 1}{a + 1}\right).$$

Exercise 9. Factor: $x^2 + 2ab - b^2 - a^2$.

Exercise 10. 1998 AHSME. Simplify the following expression completely:

$$\frac{bx(a^2x^2 + 2a^2y^2 + b^2y^2) + ay(a^2x^2 + 2b^2x^2 + b^2y^2)}{bx + ay}.$$

Exercise 11. 2007 NAMCLO. Consider these phrases in Ancient Greek (in a Roman-based transcription) and their unordered English translations (Note: the letter \bar{o} stands for a long o):

- | | |
|----------------------------|----------------------------------|
| (A) ho tōn hyiōn dulos | (1) the donkey of the master |
| (B) hoi tōn dulōn cyrioi | (2) the brothers of the merchant |
| (C) hoi tu emporu adelphoi | (3) the merchants of the donkeys |
| (D) hoi tōn onōn emporoi | (4) the sons of the masters |
| (E) ho tu cyriu onos | (5) the slave of the sons |
| (F) ho tu oicu cyrios | (6) the masters of the slaves |
| (G) ho tōn adelphōn oicos | (7) the house of the brothers |
| (H) hoi tōn cyriōn hyioi | (8) the master of the house |

Match the Greek sentences with its English translations. Translate into Ancient Greek: the houses of the merchants; the donkeys of the slave.

Exercise 12. Given that $x^3 - y^3 = a$, and $x - y = a$, where $a \neq 0$, express $x + y$ in terms of a .

Exercise 13. The number 1000...0001 has 2015 zeros. Prove that this number is composite.

Exercise 14. Alice wrote 5 integers on the board that are coefficients and roots of a quadratic equation. Bob erased one of the integers leaving the numbers 2, 3, 4, and -5 . Find the number that was erased.