

# Coins

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Question: How many sides does a box have?

Answer: Two — the inside and the outside.

## Class Discussion

Square breathing. Review counting information.

## Warm-up

**Exercise 1.** Do the following standard phrases make sense?

- I fell head over heels.
- I could care less.
- I turned my life around 360 degrees.

**Exercise 2.** You have 2 glasses of the same size, one with coffee, the other with milk. First, take a spoonful of coffee, add it to milk, and mix well. Then, take a spoonful (same amount as before) of this mixture, add it back to coffee, and mix well. Is there more milk in the coffee now, or more coffee in the milk?

**Exercise 3. 2015 Moscow Olympiad. 7th grade.** 25 camels of different heights are participating in a height competition. The jury needs to give out the top three prizes. In one round 5 camels come to the stage and jury can see their relative heights. How should the jury run the competition to minimize number of rounds? What is the smallest number of rounds?

## Counting Information

**Exercise 4.** An English-Russian dictionary has 80 pages with 50 words on each page. Mike opened the dictionary on a random page and found a random word on it. Can Pete guess the word using 13 yes-or-no questions? Can he do better?

**Exercise 5.** Mike thinks of two numbers from 1 to 10 inclusive. One number is odd, the other is even. Can Pete guess the numbers using 4 yes-or-no questions?

**Exercise 6.** We have a simple balancing scale with 2 cups (and no extra weights), and a set of  $N$  coins, which all look identical. We know that there is exactly one fake coin among them, and it is a little bit lighter than a genuine coin. All genuine coins weight the same. For which  $N$  you can always find the fake coin in two weighings? What about three weighings? What about  $k$  weighings?

**Exercise 7.** You have a balance scale and 13 coins, 1 of which is counterfeit. The counterfeit coin weighs less or more than the other coins. Can you determine the counterfeit in 3 weightings? You do not have to tell if it is heavier or lighter.

**Exercise 8.** Mike thought of one out of three numbers: 1, 2, or 3. He is allowed to answer “yes,” “no,” or “I don’t know.” Can Pete guess the number in one question?

## Competition Practice

**Exercise 9. HMNT 2008.** What is the largest  $x$  such that  $x^2$  divides  $24 \cdot 35 \cdot 46 \cdot 57$ ?

**Exercise 10. USAMO 2000.** Find the smallest positive integer  $n$  such that if  $n$  squares of a  $1000 \times 1000$  chessboard are colored, then there will exist three colored squares whose centers form a right triangle with sides parallel to the edges of the board.

**Exercise 11. IMO 1976.** Determine, with proof, the largest number which is the product of positive integers whose sum is 1976.