

Sequences

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March 17, 2014

Question: How do you keep a programmer in the shower all day?

Answer: Give him a bottle of shampoo which says "lather, rinse, repeat."

Class Discussion

Sequences. Continuing sequences. Complexity of a sequence.

Warm-Up

Exercise 1. In how many ways can you rearrange letters of my last name — Khovanova?

Exercise 2. The day before yesterday I was 25 and the next year I will be 28. (This is true only one day in a year.) When was I born?

Exercise 3. What mathematical symbol can be put between 5 and 9, to get a number bigger than 5 and smaller than 9?

Exercise 4. What is the next term of the following sequences:

- 1, 4, 9, 16, 25,
- 1, 3, 6, 10, 15, 21,
- 1, 1, 2, 3, 5, 8, 13,
- 2, 7, 1, 8, 2, 8, 1, 8, 2,
- 1, 2, 3, 2, 1, 2, 3, 4, 2, 1, 2,
- 1, 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 2, 3, 3, 4, 1,
- 3, 3, 5, 4, 4, 3, 5, 5, 4, 3,

- 8. 1, 3, 4, 7, 11, 18, 29, 47,
- 9. 1, 11, 21, 1211, 111221, 312211, 13112221,
- 10. 3, 6, 11, 18, 27, 38, 51,
- 11. 1, 2, 6, 20, 70, 252, 924, 3432, 12870, 48620,
- 12. 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9,

Competition Practice

Exercise 5. Simplify:

$$\frac{\sqrt{x} + 1}{x\sqrt{x} + x + \sqrt{x}} \div \frac{1}{x^2 - \sqrt{x}}.$$

Exercise 6. 2003 AMC 10A, Problem 21. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on a tray. How many different assortments of six cookies can be selected?

Exercise 7. 2003 AMC 10B, Problem 10. Nebraska, the home of AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by 4 digits. Each new license plate consists of three letters followed by three digits. By how many times is the number of license plates increased?

Exercise 8. 2003 AMC 10B, Problem 16. A restaurant offers three deserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a desert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003, which is not a leap year?

Exercise 9. 2004 AMC 10A, Problem 12. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties and any collection of condiments. How many kinds of hamburgers can be ordered?

Challenge Problems

Exercise 10. In how many ways can you tile a 2 by 10 rectangle by 2×1 dominoes?