

Review

Tanya Khovanova

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Teacher: If you were to add 87,326 and 139,561 — and then multiply by 8, add 9,182 and divide by 7, what would you get?

Student: The wrong answer.

Warm-Up

Exercise 1. Decode (Every letter represents a different digit):

$$A + AB + ABC = BCB.$$

Exercise 2. What is the smallest number of pencils that you need to take in the dark from a box containing 7 red pencils and 5 blue ones, so that you will have at least two red and three blue pencils?

Exercise 3. A caterpillar started climbing to the top of a tree on a Monday morning. She travels up for half a day, then she sleeps for twelve hours. During the day time she climbs 4 meters, and she slides 3 meters when she sleeps. When will she be at the top, if the tree is 12 meters high?

Exercise 4. Insert pluses in a row of 20 fives: 555...55, so that the sum is 1000.

Exercise 5. On Monday the baby said: A, on Tuesday AU, on Wednesday AUUA, on Thursday AUUAUAAU. What will he say on Saturday?

Exercise 6. Mike said to Bob: “Give me two dollars, and we will have the same amount of money.” Bob replied: “No, you give me two dollars and I will have twice as much money as you.” How much money does each of them have?

Exercise 7. I can fit two pancakes into my pan at the same time. I need to fry each pancake for two minutes on each side. What is the minimum time I need for frying 3 pancakes?

Competition Practice

Exercise 8. 2002 AMC 12A. For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \leq m + n$?

Exercise 9. Calculate:

$$\frac{2\frac{3}{4}/1.1 + 3\frac{1}{3}/\frac{5}{7}}{2.5 - 0.4 \cdot 3\frac{1}{3}} - \frac{(2\frac{1}{6} + 4.5) \cdot 0.375}{2.75 - 1\frac{1}{2}}.$$

Exercise 10. 2002 AMC 10A. Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

Exercise 11. 2002 AMC 12B. The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \dots, 999999999\}$ is a 9-digit number M , all of whose digits are distinct. What digits the number M does not contain?

Exercise 12. 2005 AMC 10B. A positive number x has the property that $x\%$ of x is 4. What is x ?

Challenge Problems

Exercise 13. There are 2 hourglasses measuring 7 and 11 minutes respectively. How do you measure 15 minutes? What about any number of mins?

Exercise 14. Pooh forgot to wind up his grandpa's big clock and it stopped while he was sleeping. He knows that Piglet has a clock. How can he put the clock to the right time without moving any clocks around? We assume that they do not have telephones or radios.

Exercise 15. There are three athletes (Alex, Brook and Chris) and their individual coaches (Murphy, Newlyn and Oakley) standing on the shore. No coach trusts their athlete to be anywhere near any other coach unless they are also with them. There is a boat that can hold a maximum of two persons. How can the six people get across the river?