

# Miscellaneous Probability

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## Class Discussion

Sicherman dice, non-transitive dice.

## Warm Up

**Exercise 1.** What is the probability of getting five Mondays in a 31-days month?

**Exercise 2.** Use arithmetic operations to express 6 using three identical digits. These are 10 puzzles in one. For example, using 2 you can express 6 as  $2 + 2 + 2$ . Now do it using 0,1,3,4,5,6,7,8,9.

**Exercise 3.** You have three bags, each containing two marbles. Bag 1 contains two white marbles, bag 2 contains two black marbles, and bag 3 contains one white marble and one black marble.

You pick a random bag and take out one marble without seeing the other marble. Given that this is a white marble, what is the probability that the remaining marble from the same bag is also white?

**Exercise 4.** What is the probability that a random 2-digit number has 5 as a sum of digits?

## Competition practice

**Exercise 5. Math Kangaroo.** The children A, B, C and D made the following assertions below. Is it possible to determine how many of the children were telling the truth? If so, how many?

- A: "B, C and D are girls"
- B: "A, C and D are boys"
- C: "A and B are lying"
- D: "A, B and C are telling the truth"

**Exercise 6. Fifth Kolmogorov Student Olympiad in Probability Theory.** In a hand are six blades of grass next to each other pointing in the same direction. The tops of the blades are randomly collected into pairs that are linked together, and likewise with the bottoms of the blades. What is the probability that as a result of this operation, the six blades will be connected in a single ring?

## Challenge Problems

**Exercise 7.** What is the probability of choosing the correct answer at random from the options below?

- a)  $1/4$ , b)  $1/2$ , c) 1, d)  $1/4$ .

**Exercise 8. Famous interview question.** Let's play a game of Russian roulette. You are tied to your chair and can't get up. Here's a revolver with six chambers, all empty. Now watch me as I put two bullets in the revolver adjacent to each other. I close the cylinder and spin it. I put a gun to your head and pull the trigger. Click. The chamber was empty. Lucky you! Now I'm going to pull the trigger one more time. Which would you prefer, that I spin the cylinder first, or that I just pull the trigger?

**Exercise 9.** Bruna was first to arrive at a 100 seat theater. She forgot her seat number and picks a random seat for herself.

After this, every single person who get to the theater sits on his/her seat if its available, else chooses any available seat at random. Neymar is last to enter the theater and 99 seats were occupied.

Whats the probability what Neymar gets to sit in his own seat ?

**Exercise 10. Chomp.** Chomp is a two-player strategy game played on a rectangular chocolate bar made up of smaller square blocks (cells). The players take it in turns to choose one block and "eat it" (remove from the board), together with those that are below it and to its right. The top left block is "poisoned" and the player who eats this loses.

Prove that the first person always wins.