AMC Preparation. Mental Arithmetic

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— Do you know that 67% of people are not capable of doing simple arithmetic?
— I belong to the other 23%.

Class Discussion

- Fast multiplication by 5, 25, 4, 8, 11, 18, 9, 99, 22.
- Fast division: 4, 5, 8.
- Squares of numbers ending in 5. Fast squaring. Squares of numbers from 26 through 50. Consecutive squares. Product of numbers of the form $9x$.
- A magic trick when I guess the first digit of a product of many digits and I am told other digits in random order.

Warm-Up

Exercise 1. Pooh poured himself a cup of coffee, then drank 1/2 of the cup, then topped it off with milk, stirred it, then drank 1/3 of the cup, then topped it off with milk, stirred it, then drank 1/6 of the cup, then topped it off with milk, stirred it, drank up the whole cup. Find the ratio of the amounts of coffee and milk that he had drunk.

Mental Arithmetic and Competition Practice

Exercise 2.
$$\frac{(7 - 6.35)/6.5 + 9.9}{(1.2/36 + 1.2/0.25 - 1\frac{5}{16})/\frac{169}{24}}$$

Exercise 3.

$$\left(\frac{7/9 - 47/72}{1.25} + \frac{6/7 - 17/28}{0.358 - 0.108}\right) \times 1.6 - \frac{19}{25}$$

Exercise 4. There are 4 people in a family. If the daughter's salary is doubled, then the total family income will increase by 5%. If, instead, Mom's salary is doubled, it will increase by 20%. If Dad's salary is doubled, it will increase by 20% too. By what percentage will the total income increase if Grandpa's pension is doubled?

Exercise 5. 1999 AMC 8, Problem 1. $(6 \, ? \, 3) + 4 - (2 - 1) = 5$. By what operation the question mark should be replaced with?

Exercise 6. 2007 AMC 8, Problem 7. The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

Exercise 7. 2002 AMC 10A, Problem 6. Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

Exercise 8. 2002 AMC 10A, Problem 12. Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

Challenge Problems

Exercise 9. Given 99 blank cards, we first write the numbers 1, 2, ..., 99 on them, one number per card. We then shuffle the cards, turn them upside down, and again write the numbers 1, 2, ..., 99 on them. Then, for each card, we add the 2 numbers written on it, and then multiply these 99 new numbers. Prove that the result is an even number.

Exercise 10. A farmer brought 9 cows to a pasture and they ate all the grass in 6 days. If he had brought 8 cows instead, they would have eaten all the grass in 9 days. What's the maximum number of cows that can feed on this pasture forever, while the grass is growing?