

Every and There Exists

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Class Discussion

Negation. Using EVERY and THERE EXISTS.

Warm-Up

Exercise 1. Lewis Carroll. From the premises:

1. Babies are illogical;
2. Nobody is despised who can manage a crocodile;
3. Illogical persons are despised.

What can you conclude?

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Exercise 2. Write the negations of the following statements:

1. All cats like hot dogs.
2. It is not true that today is Friday.
3. Some dogs eat fish.
4. $1 + 1 = 3$.
5. x is an irrational number.

Exercise 3. Correct conclusion or not?

1. All crocodiles can fly. All giants are crocodiles. Therefore, all giants can fly.

2. Some cabbage-heads are locomotives. Some locomotives play grand piano. Therefore some cabbage-heads play grand piano.
3. No person can become a president if s/he has a red nose. Every person has a red nose. Therefore, no person can become a president.
4. All children love math. Some people who love math have 3 ears. Therefore, some children have 3 ears.

Logic

Exercise 4. Each of the 4 dwarfs — Grumpy, Happy, Sleepy and Sneezzy — either always tells the truth or always lies. We heard the following conversation: Grumpy to Happy: “You are a liar!” Sleepy to Grumpy: “You are a liar yourself!” Sneezzy to Sleepy: “They are both liars. You are a liar too.”

Who is a truth-teller and who is a liar?

Exercise 5. Mary Ann Moore’s father has a yacht and so do each of his four friends: Colonel Downing, Mr. Hall, Sir Barnacle Hood, and Dr. Parker. Each of the five also has one daughter and each has named his yacht after a daughter of one of the others. Sir Barnacle’s yacht is the Gabrielle, Mr. Moore owns the Lorna; Mr. Hall the Rosalind. The Melissa, owned by Colonel Downing, is named after Sir Barnacle’s daughter. Gabrielle’s father owns the yacht that is named after Dr. Parker’s daughter. Who is Lorna’s father?

Exercise 6. Baker, Cooper, Fletcher, Miller, and Smith live on different floors of an apartment house that contains only five floors. Baker does not live on the top floor. Cooper does not live on the bottom floor. Fletcher does not live on either the top or the bottom floor. Miller lives on a higher floor than does Cooper. Smith does not live on a floor adjacent to Fletcher’s. Fletcher does not live on a floor adjacent to Cooper’s. Where does everyone live?

Competition practice

Exercise 7. 2001 AMC 8, Problem 20. Kaleana, Quay, Marty and Shana discuss their test scores. Kaleana is the only one who shows her score to the others. Quay says, “At least two of us have the same score.” Marty says, “I didn’t get the lowest score.” Shana adds, “I didn’t get the highest

score.” Rank the scores from lowest to highest for Marty (M), Quay (Q) and Shana (S).

Exercise 8. 2003 AMC 10 A, Problem 24. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

Exercise 9. 2004 AMC 10 A, Problem 6. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters.

How many of Bertha’s daughters and granddaughters have no daughters?

Exercise 10. Mathcounts 1987, Team round. Each of the marbles A , B and C is colored one of three colors. One marble is white, one marble is red, and one marble is blue. Exactly one of the following statements is true.

- A is red.
- B is not blue.
- C is not red.

What color is marble B ?

Challenge Problems

Exercise 11. Einstein’s Puzzle. Variations of this riddle appear on the net from time to time. It is sometimes attributed to Albert Einstein and it is claimed that 98% of the people are incapable of solving it. Some commentators suggest that Einstein created such puzzles not to test out intelligence but to get rid of all the students who wanted him as an advisor. It is not likely that there is any truth to these stories. Wherever this comes from, it is a nice riddle.

Let us assume that there are five houses of different colors next to each other on the same road. In each house lives a man of a different nationality. Every man has his favorite drink, his favorite brand of cigarettes, and keeps pets of a particular kind.

1. The Englishman lives in the red house.

2. The Swede keeps dogs.
3. The Dane drinks tea.
4. The green house is just to the left of the white one.
5. The owner of the green house drinks coffee.
6. The Pall Mall smoker keeps birds.
7. The owner of the yellow house smokes Dunhills.
8. The man in the center house drinks milk.
9. The Norwegian lives in the first house.
10. The Blend smoker has a neighbor who keeps cats.
11. The man who smokes Blue Masters drinks bier.
12. The man who keeps horses lives next to the Dunhill smoker.
13. The German smokes Prince.
14. The Norwegian lives next to the blue house.
15. The Blend smoker has a neighbor who drinks water.

The question to be answered is: Who keeps fish?

Exercise 12. Lewis Carroll. From the premises:

1. The only animals in this house are cats;
2. Every animal is suitable for a pet, that loves to gaze at the moon;
3. When I detest an animal, I avoid it;
4. No animals are carnivorous, unless they prowl at night;
5. No cat fails to kill mice;
6. No animals ever take to me, except what are in this house;
7. Kangaroos are not suitable for pets;
8. None but carnivora kill mice;
9. I detest animals that do not take to me;
10. Animals, that prowl at night, always love to gaze at the moon.

What can you conclude?