Counting Information

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Class Discussion

Counting Information. Nine coins problem. 12 coins problem. One fake coin problem. Static and dynamic strategies.

Warm-Up

Exercise 1. Bob has two more sisters than brothers. How many more daughters than sons do Bob's parents have?

Exercise 2. A two digit number is summed up with its reverse. The resulting number is a square. Find all such numbers.

Counting Information

Exercise 3. I have N coins, one of them is fake and is lighter than the others. I also have a balance with two pans. I can put some number of coins into each pan, and the balance shows me which set of coins is lighter. I need to find the fake coin using the balance the fewest number of times. How many times do I need to use the balance if I have 7 coins? Show how to process any number of coins between 4 and 8 inclusive in 2 weighings.

Exercise 4. Invent a static strategy to find the fake coin (which is known to be lighter) out of 9 coins in two weighings?

Exercise 5. There are five chess players of different strengths. If two of them play, the stronger one always wins. What is the minimum number of games they need to play for us to determine the order of their strengths?

Competition Practice

Exercise 6. HMNT 2008. Al has a rectangle of integer side lengths a and b, and area 1000. What is the smallest perimeter it could have?

Exercise 7. HMNT 2008. How many numbers between 1 and 1,000,000 are perfect squares but not perfect cubes?

Exercise 8. HMNT 2008. A triangle has sides of length 9, 40, and 41. What is its area?

Exercise 9. HMNT 2008. Sarah is deciding whether to visit Russia or Washington, DC for the holidays. She makes her decision by rolling a regular 6-sided die. If she gets a 1 or 2, she goes to DC. If she rolls a 3, 4, or 5, she goes to Russia. If she rolls a 6, she rolls again. What is the probability that she goes to DC?

Challenge Problems

Exercise 10. You have a balance scale and 12 coins, one of which is counterfeit. The counterfeit weighs less or more than the other coins. Can you determine the counterfeit in 3 weighings, and tell if it is heavier or lighter using a static strategy?