Bases

Tanya Khovanova

November 4, 2013

Class Discussion

A static strategy for the Nine-coins problem, ternary numbers. Divisibility in bases: divisibility by $n/2$, by $n-1$, by $n+1$. Sum and multiply numbers within a base. Find $11_6 \times 11_6$, same for 11_6^3. Hexadecimals, conversion.

Warm-Up

Exercise 1. from Raymond Smullyan's book. Twenty-four red socks and twenty-four blue socks are lying in a drawer in a dark room. What is the minimum number of socks I must take out of the drawer which will guarantee that I have at least two socks of the same color?

Exercise 2. from Raymond Smullyan's book. Suppose some blue socks and the same number of red socks are in a drawer. Suppose it turns out that the minimum number of socks I must pick in order to be sure of getting at least one pair of the same color is the same as the minimum number I must pick in order to be sure of getting at least two socks of different colors.

How many socks are in the drawer?

Bases

Exercise 3. If only DEAD people understand hexadecimal, how many people understand hexadecimal?

Exercise 4. What is the smallest number of weights that you need to weigh any number of grams from 1 to 100 on a balance scale if you are allowed to put weights only on the left pan? What if you are allowed to put weights on both pans?

Exercise 5. The room you are in has three switches, which operate three light bulbs in the basement. You can't see the basement from any other room. You need to deduce which switch operates which light bulb. You may only enter the basement once. How is this possible?

How many switches and light bulbs can you match if you are allowed to go to the basement twice?

Exercise 6. Describe all the bases in which a number is divisible by 2 if and only if its sum of digits is divisible by 2. Do the same thing for any number $m > 1$.

Competition Practice

Exercise 7. HMNT 2009. Given that $a+b+c = 5$ and that $1 \leq a, b, c \leq 2$, what is the minimum possible value of $\frac{1}{a+b} + \frac{1}{b+c}$?

Exercise 8. HMNT 2009. What is the period of the function $f(x) = \cos(\cos(x))$?

Exercise 9. HMNT 2009. How many subsets A of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ have the property that no two elements of A sum to 11?

Exercise 10. HMNT 2009. A polyhedron has faces that are all either triangles or squares. No two square faces share an edge, and no two triangular faces share an edge. What is the ratio of the number of triangular faces to the number of square faces?

Challenge Problems

Exercise 11. Prove that you can choose a set of 2^k numbers from the numbers 0, 1, 2, ..., $3^k - 1$ so that no number in the set is an average of two numbers in the set.