Factorials

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November 5, 2012

Class Discussion

Marshmallow Test.

Warm-Up

Exercise 1. I owe my friend money. Today, I paid back \$100, and I still owe as much as I would still owe if I had paid as much as I still owe. What was my original debt?

Factorials

Exercise 2. Find the smallest n such that $n!$ is divisible by 990.

Exercise 3. Can $n!$ end with exactly 5 zeros?

Exercise 4. Prove that 100! can't be a perfect square. For which n, $n!$ can be a perfect square?

Exercise 5. What is the smallest positive integer that is not a factor of 50!?

Exercise 6. The numbers 1, 2, ..., 9 are divided into three groups. Prove that the product of numbers in one of the groups is at least 72.

Exercise 7. What number is greater: 200! or 100^{200}?

Competition Practice

Exercise 8. HMNT 2008. General Round. How many integers between 2 and 100 inclusive cannot be written as $m \cdot n$, where m and n have no common factors and neither m nor n is equal to 1? Note that there are 25 primes less than 100.

Exercise 9. HMNT 2008. General Round. Find the product of all real x for which $2^{3x+1} - 17 \cdot 2^{2x} + 2^{x+3} = 0$.

Challenge Problems

Exercise 10. Find all the numbers n such that $(n-1)!$ is not divisible by n^2.