Test

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Your name:

Your grade:

Exercise 1. 1 point. 100 pounds of cucumbers, that were 99% water, got a bit dehydrated, and became 98% water. What is their weight now?

Answer:

Exercise 2. 1 point. The bad guy discovered that Tanya's password starts with 2 capital letters followed by 2 digits, then by 2 lower-case letters followed by 2 characters at the end that could be either digits or capital letters. How many possibilities are there for Tanya's password? Provide a formula.

Answer:

Exercise 3. 1 point. Tanya wants to pick three HMNT teams of six people each from a total of 43 students. In how many ways can she do that? Provide a formula.

Answer:

Exercise 4. 1 point. How many ways are there to place two white bishops on two white cells of a chessboard, so they do not attack each other?

Answer:

Exercise 5. 1 point. How many zeros are there at the end of the decimal expansion of 149! ?

Answer:

Exercise 6. 1 point. How many distinct four-digit numbers are divisible by 3 and have 29 as their last two digits?

Answer:

Exercise 7. 1 point. Find $\sin 18^\circ$.

Answer:

Exercise 8. 1 point. Find the smallest base such that a number in this base is divisible by 7 iff (if an only if) the sum of the digits is divisible by 7 and a number in this base is divisible by 5 iff the last two digits form a number divisible by 5.

Answer:

Exercise 9. 1 point. Compute the probability that a randomly chosen three-digit number is a palindrome.

Answer:

Exercise 10. 2 points. Johnny is typing all the integers from 1 to infinity, in order. The 4 on his computer is broken however, so he just skips any number with a 4. What's the 2012th number he types?

Answer:

Exercise 11. 2 points. A survey asking people how happy they are shows that college math professors are the happiest among all profession. Therefore, to be happy you should become a college math professor. Is the conclusion correct? Explain.

Answer:

Exercise 12. 2 points. You have a piece of chocolate. You are allowed to divide it into four pieces or eight pieces. Then you can choose a piece and repeat the procedure again. What is the largest number of pieces that is impossible to get? Explain.

Answer:

Exercise 13. 3 points. Knights always tell the truth and Knaves always lie. You find yourself on an Island full of Knights and Knaves who have been stranded there (mainly because the armor makes it difficult to swim the 100 meters or so to the mainland.)

You meet four inhabitants: Ted, Carl, Alice and Rex. Ted says that only a knave would say that Rex is a knave. Carl claims, 'Ted would tell you that Rex is a knight.' Alice claims that Ted and Rex are both knights or both knaves. Rex says that at least one of the following is true: that Carl is a knave or that Alice is a knave. Sort this out... Explain.

Answer:

Exercise 14. 3 points. We have N coins that look identical, but we know that exactly one of them is fake. The genuine coins all weigh the same. The fake coin is either lighter or heavier than a real coin. We also have a balance scale. Unlike in classical math problems where you need to find the fake coin, in this problem your task is to figure out whether the fake coin is heavier or lighter than a real coin. Your challenge is that you are only permitted to use the scale twice. Find all numbers N for which this can be done. Explain.

Answer:

Exercise 15. 4 points. A parallelogram with sides 2 and 4 has a diagonal drawn. Its length is 3. Each resulting triangle has a circle inscribed in it. Find the distance between the centers of these circles. Explain.

Answer: