

# Triangle's Area

Tanya Khovanova

April 23, 2012

## Class Discussion

Heron's formula: if  $s$  is a semiperimeter of a triangle with sides  $a$ ,  $b$ ,  $c$ , then its area is  $\sqrt{s(s-a)(s-b)(s-c)}$ .

If  $A$  is at the origin, then the area of the triangle  $ABC$  can be computed using coordinates of  $B$  and  $C$ :  $\frac{1}{2}|x_B y_C - x_C y_B|$ .

## Warm-Up

**Exercise 1.** My dining room chandelier has 5 light bulbs in it. During a storm two of them burned out. How many light bulbs are in the chandelier now?

**Exercise 2.** One hundred percent of the fish in a pond are goldfish. I take 10% of the goldfish out of the pond. What percentage of goldfish does the pond now contain?

**Exercise 3.** It takes 1629 digits to number the pages of a book. How many pages does the book have?

## Area of a triangle

**Exercise 4.** Does there exist a triangle with altitudes of lengths 1, 2, and 3?

**Exercise 5.** Among triangles with given sides  $a$  and  $b$  which has the largest area?

**Exercise 6.** Every diagonal of a pentagon divides the pentagon into a quadrilateral and a triangle of area 1. Find the area of the pentagon.

**Exercise 7.** Given three angles of a triangle  $A$ ,  $B$ ,  $C$ , and one side  $c$ , find the area.

**Exercise 8.** In triangle  $ABC$  we know three sides:  $AB = 26$ ,  $BC = 30$  and  $AC = 28$ . Find the area of a triangle between the bisector and the altitude passing through  $B$ .

### Challenge Problems

**Exercise 9.** Once upon a time there was a land where the only antidote to a poison was a stronger poison. In this land, a dragon challenges a king to a duel and the king accepts. The rules of the duel are as follows: Each dueler has a cup of poison. First, they exchange cups. Then, each dueler drinks half of the poison in front of them. After that, they exchange again and drink their own poisons. The survivor wins. The king knows that the dragon will use a stronger poison than any the king will ever make. How can the king win the duel?

**Exercise 10.** Let  $ABC$  be an isosceles triangle ( $AB = AC$ ) with  $\angle BAC = 20^\circ$ . Point  $D$  is on side  $AC$  such that  $\angle CBD = 50^\circ$ . Point  $E$  is on side  $AB$  such that  $\angle BCE = 60^\circ$ . Find the measure of  $\angle CED$ .