Fancy Number Systems

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Class Discussion

Factoradix — factorial number system. Zeckendorf presentation.

Warm-Up

Exercise 1. Two wallets contain a total of two coins, and one wallet contains twice as many coins as the other. How could it be?

Exercise 2. The son of the professor's father talks to the father of the professor's son? Can it be if the professor doesn't participate in the conversation?

Exercise 3. In a ten-story building, one person lives on the first floor, two people on the second floor, and so on: ten people on the tenth floor. At which does floor the elevator stop most often?

Number Systems

Exercise 4. Describe all bases, such that a number in this base is even iff its sum of digits is even.

Exercise 5. Prove that any integer A can be represented as $a_0 + a_1 2 + a_2 2^2 + a_n 2^n$, where a_k is 0, 1 or -1, and $a_k a_{k+1} = 0$, for $0 \leq k \leq n - 1$. Prove that this representation is unique.

Exercise 6. One person thought of ten natural numbers: x_1, x_2, ..., x_{10}. The second person tries to guess these numbers. The second person is allowed to ask questions of the form, "How much is $a_1 x_1 + a_2 x_2 + \ldots + a_{10} x_{10}$, where a_i are natural numbers. What is the smallest number of question that can

guarantee that the second person can figure out all the numbers that were thought of.

Competition Practice

Exercise 7. HMNT 2008, Guts round 14 points. Johnny the grad student is typing all the integers from 1 to ∞, in order. The 2 on his computer is broken however, so he just skips any number with a 2. What's the 2008th number he types?

Exercise 8. HMNT 2009, Guts round 7 points. Daniel wrote all the positive integers from 1 to n inclusive on a piece of paper. After careful observation, he realized that the sum of all the digits that he wrote was exactly 10,000. Find n.

Challenge Problems

Exercise 9. I thought of a number between 1 and 144 inclusive. You are allowed to pick a set of numbers and ask me if my number is in the set. Your questions are not free, you have to pay $2 for a "yes" and $1 for a "no." What is the smallest amount of money you need to guarantee that you can guess my number?

Exercise 10. You need to choose ten weights that weigh an integer number of grams, so that you can weigh any number between 1 and 55 grams inclusive even if you lose one of the weights.