

# Extra Problems. VII. HMMT 2009 Algebra

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## 3 points

If  $a$  and  $b$  are positive integers such that  $a^2 - b^4 = 2009$ , find  $a + b$ .

## 4 points

Let  $a, b$ , and  $c$  be the 3 roots of  $x^3 - x + 1 = 0$ . Find  $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}$ .

## 4 points

Suppose  $a, b$  and  $c$  are integers such that the greatest common divisor of  $x^2 + ax + b$  and  $x^2 + bx + c$  is  $x + 1$  (in the ring of polynomials in  $x$  with integer coefficients), and the least common multiple of  $x^2 + ax + b$  and  $x^2 + bx + c$  is  $x^3 - 4x^2 + x + 6$ . Find  $a + b + c$ .