Extra Problems. II.

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Functions with a Special Property

Find all functions $F(x) : \mathbb{R} \to \mathbb{R}$ having the property that for any x_1 and x_2 the following inequality holds:

$$F(x_1) - F(x_2) \leq (x_1 - x_2)^2. \tag{1}$$

Pair Sharing Prime Factors

Consider the set of pairs of distinct integers A and B, such that the set of prime factors of A is the same as the set of prime factors of B, and such that the sets of prime factors of $A - 1$ and $B - 1$ are likewise equal. Is this set of pairs finite or not?

A Perpendicular

There is a circle in the plane with a drawn diameter. Given a point, draw the perpendicular from the point to the diameter using only a straightedge. Assume the point is neither on the circle nor on the diameter line.