

# Extra Problems. I.

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## Fibonacci Numbers

Prove that for any integer  $n$  there exists a Fibonacci number divisible by  $n$ .

## Zeroes and Ones

Prove that for any integer  $n$  there exists an integer consisting only of ones and zeroes that is divisible by  $n$ .

## Three Sections

Divide your paper into three sections. In one section write the number 2, in a different section write the number 3. Thereafter, if two numbers are in distinct sections, write their product in the third section. For example,  $6 = 2 \times 3$  goes into the initially empty section, then  $12 = 2 \times 6$  goes into the same section with 3, etc.

- Does the same number ever end up in two distinct sections?
- Obviously all numbers will be of the form  $2^k 3^j$  with  $j, k \geq 0$ . Which of these numbers never appear?
- Describe numbers that appear in every section.