

# Fair Division

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## Class Discussion

Dividing inheritance.

## Warm-Up

**Exercise 1.** Captain Cook made three voyages to the Pacific Ocean and died during one of them. Which one?

**Exercise 2.** If eggs are twelve cents a dozen, how much 100 eggs would cost?

**Exercise 3.** There are six glasses in a row. The first three are full of water, and the next three are empty. By touching only one glass how can you make them alternate between full and empty?

**Exercise 4.** A mile-long train is moving at sixty miles an hour when it reaches a mile-long tunnel. How long does it take the entire train to pass through the tunnel?

## Fair Division

**Exercise 5.** A fair division procedure is called *envy-free* if after the division no one wants to swap his/her part for someone else's. For three people it means each person believes s/he got at least one third and no one got more than s/he. Suggest an envy free division strategy for three people.

**Exercise 6.** Alice and Bob are divorcing. They want to divide everything fairly. They own \$500,000 in cash. In addition, they own a house. Bob values the house at its market price of \$400,000. Alice loves the neighborhood and

hates moving, so she values the house at \$440,000. They also own a vacation house that Bob uses for his fishing trips. Alice values the vacation house at \$200,000, while Bob at \$240,000. They also own a portrait of Alice's grandfather, which Bob values at 0, while Alice values at \$10,000. Suggest how they can divide their stuff fairly.

## Competition Practice

**Exercise 7.** Find a four digit number with distinct digits, such that the multiplication by 4 reverses the digits.

**Exercise 8. ARML.** How many zeroes do we write when we write all the integers from 1 to 256 in binary?

**Exercise 9. MATHCOUNTS.** The Sagebrush student council has 6 boys and 6 girls as class representatives. Two committees, each consisting of 2 boys and 2 girls, are to be created. If no student can serve on both committees, how many different combinations of committees are possible?

**Exercise 10.** How many positive integers less than 1000 are there such that the digits are in strictly increasing order?

**Exercise 11.** How many pairs of diagonals of a regular hexagon are parallel? Same question for an octagon and a decagon. Can you continue the sequence?

**Exercise 12.**  $N$  different circles are drawn in a plane. What is the maximum number of different points at which the circles can meet?

## Challenge Problem

**Exercise 13.** The sultan decides to torture his hundred wise men again. He has an unlimited supply of red and blue hats. Tomorrow he will pile two hats randomly on each wise man's head. Each wise man will see the colors of other people's hats, but not the colors of his own hats. The men are not allowed to pass any info to each other. At the sultan's signal each has to write the number of blue hats on his head. If they are all correct, all of them survive. If at least one of them is wrong, all of them die. What should be their strategy to maximize their chance of survival?