

# Translations

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## Class Discussion

M.C. Escher. Translations. Logic Review.

Given a vector  $a$ , the slide of a figure in the direction of  $a$  at a distance equal to the length of  $a$  is called a *translation*.

**Theorem.** A translation  $F'$  of a figure  $F$  is congruent to  $F$ .

## Warm Up

**Exercise 1.** Three horses were galloping at 27 miles per hour. What was the speed of one horse?

**Exercise 2.** Every point on the plane is either black or white. Prove that there are two points of the same color at a distance of one meter.

## Problem Set

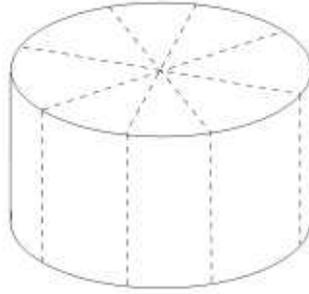
**Exercise 3.** There are  $n$  points on the plane. Prove that the shortest zig-zag line connecting all of them doesn't intersect itself.

**Exercise 4.** Prove that a translation of a circle is a circle.

**Exercise 5.** Given two circles  $R$  and  $S$  and a segment  $AB$ , build a segment that is equal and parallel to  $AB$  with the end points belonging to the given circles.

**Exercise 6.** Find the locus of points  $M$  the sum of whose distances from two given lines  $l_1$  and  $l_2$  is equal to a given value  $a$ .

**Exercise 7.** It's eazy to see that a cylinder of cheese can be cut into eight pieces with four straight cuts. What is the maximum number of pieces you



can get with three straight cuts?

**Exercise 8.** Can a cube of cheese three inches on a side be cut into 27 one-inch cubes with five straight cuts? What if one can move the pieces prior to cutting?

**Exercise 9.** In this problem, “truthtellers” always tell the truth and “liars” always lie. In (1)-(3), all persons are either truthtellers or liars.

1. There are two persons, A and B. A says, “At least one of us is a liar.” What are A and B?
2. A says, “Either I am a liar or B is a truthteller.” What are A and B?
3. Now we have three persons, A, B, and C. A says, “All of us are liars.” B says, “Exactly one of us is a truthteller.” What are A, B, and C?
4. Now we have a third type of person, called “normal,” who sometimes lies and sometimes tells the truth. A says, “I am normal.” B says, “That is true.” C says, “I am not normal.” Exactly one of A, B, C is a truthteller, one is a liar, and one is normal. What are A, B, and C?