Rotations

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Class Discussion

M.C. Escher. Rotations.

Given point O and angle α, for any point A we can build point A' which is the *rotation* of A around point O with the angle α. The point A' satisfies two conditions: $\angle AOA' = \alpha$ and $OA = OA'$.

Figure F' which is the set of all points that are rotations of all the points of figure F around point O with the angle α is by definition a rotation of figure F around point O with the angle α.

Warm Up

Exercise 1. A certain sheik named Hassan had eight horses. Four of them were white, three were black, and one was brown. Assuming now that Hassan's horses can talk, how many of them can each say that it is the same color as another one of Hassan's horses?

Exercise 2. If a certain plant were three feet taller, then it would be twice as tall as it would be if it were half a foot less. How tall is the plant?

Problem Set

Exercise 3. Segments AB and CD are of equal length and they are not parallel and do not belong to the same line. Prove that there exists a unique rotation that moves AB to CD, such that point A moves to C and point B moves to D.

Exercise 4. Given three parallel lines a, b and c, build an equilateral triangle ABC with vertices A, B and C belonging to the lines a, b and c correspondingly (see Figure 1).

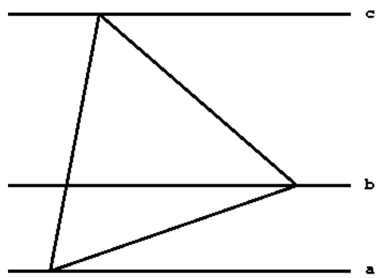

Figure 1:

Exercise 5. For the capital letters of the English alphabet find the letters that have a point of symmetry.

Exercise 6. Find point M inside a triangle ABC such that the sum of the distances from this point to the vertices of the triangle is minimal.

Exercise 7. Suppose that someone put down a copper coin, a silver coin, and a gold coin and asked you to make a statement, with the understanding that if your statement is true, then you will be given one of the three coins, but if your statement is false you will be given no coin. What statement could you make that would guarantee that you would get the gold coin?

Exercise 8. HMNT 2008. Al has a rectangle of integer side lengths a and b, and area 1000. What is the smallest perimeter it could have?

Exercise 9. HMNT 2008. $ABCDE$ is a regular pentagon inscribed in a circle of radius 1. What is the area of the set of points inside the circle that are farther from A than they are from any other vertex?

Exercise 10. Spivak middle school Olympiad. A rectangle is cut into several rectangles with integer perimeters (in centimeters). Can we conclude that the perimeter of the starting rectangle is an integer (in centimeters)?

Exercise 11. Spivak middle school Olympiad. In an acute triangle the smallest angle is 1/5 of the largest angle. Find the angles, if all of them are integer degrees and distinct.