Divisibility Rules

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Class Discussion

A simple magic trick: take a number, reverse it, subtract the difference, sum the digits, I guess the sum.

Divisibility rules: 2,3,4,5,6,8,9,10,11,12.

Why 1001 is useful?

Explain about 1001 and divisibility by 7, 11 and 13.

Puzzle together: Create the largest number containing all of the digits from 0 to 9 once and which is divisible by 36.

Explain the trick.

The second trick at the end.

Warm Up

Exercise 1. Mike and Tom went to a yard sale and wanted to buy a Yoda toy. Mike needed 10 more cents to buy the toy and Tom needed 1 more cent. They put their money together and they still didn't have enough. How much was Yoda?

Exercise 2. Bob has two more sisters than brothers. How many more daughters than sons do Bob's parents have?

Problem Set

Exercise 3. Can you replace the stars in the equation $1 \star 2 \star 3 \star ... \star 10 = 0$ with pluses and minuses to get a correct equality?

Exercise 4. Prove that the number of different divisors of n (including 1 and n) is odd if and only if n is a square.

Exercise 5. A number is written with 300 ones and all other digits are zeroes. Can this number be a square?

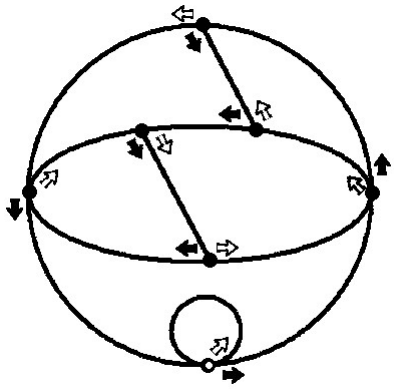
Exercise 6. A two digit number is summed up with its reverse. The resulting number is a square. Find all such numbers.

Exercise 7. What two numbers, neither of them containing zeros, can be multiplied together to make 5,000,000,000?

Exercise 8. What is the last digit of 7^{2009}?

Exercise 9. How many zeroes does 100! have at the end?

Exercise 10. Write down all the natural numbers in a row: 12345678910111213 .... What digit is on the 1000-th place?



Exercise 11. I have attached a picture of a graph.
Write down a number n. Start at the small white node at the bottom of the graph. For each digit d in n, follow d black arrows in a succession, and as you move from one digit to the next, follow 1 white arrow. For example, if $n = 325$, follow 3 black arrows, then 1 white arrow, then 2 black arrows, then 1 white arrow, and finally 5 black arrows.

If you end up back at the white node, n is divisible by 7. Why does this procedure work?