Combinatorics. Other Homework problems

Tanya Khovanova

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There are three kinds of people in the world: those who can count and those who can't.

Finish the problems from the class handout.

Competition practice

Exercise 1. 2006 AMC 12B, Problem 7. Mr. and Mrs. Lopez have two children. When they get into their family car, two people sit in the front, and the other two sit in the back. Either Mr. Lopez or Mrs. Lopez must sit in the drivers seat. How many seating arrangements are possible?

Exercise 2. 2006 AMC 12B, Problem 9. How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?

Exercise 3. 2006 AMC 12B, Problem 18. An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?

Exercise 4. 2006 AMC 12A, Problem 25. How many non-empty subsets S of $\{1, 2, 3, \ldots, 15\}$ have the following two properties?

1. No two consecutive integers belong to S

2. If S contains k elements, then S contains no number less than k.

Exercise 5. Calculate:

$$\frac{\left(5\frac{4}{45} - 4\frac{1}{6}\right)/5\frac{8}{15}}{\left(4\frac{2}{3} + 0.75\right) \cdot 3\frac{9}{13}} \cdot 34\frac{2}{7} + \frac{0.3/0.01}{70} + \frac{2}{7}.$$

Challenge Problems

Exercise 6. How many subsets of the set $\{1, 2, 3, \ldots, 11\}$ can you form in such a way that the subsets do not contain any two consecutive numbers?

Exercise 7. In how many ways can you roll two dice (I want you to count a roll of a 5 and a 6 the same as a 6 and a 5)? three dice?