Triangle Points

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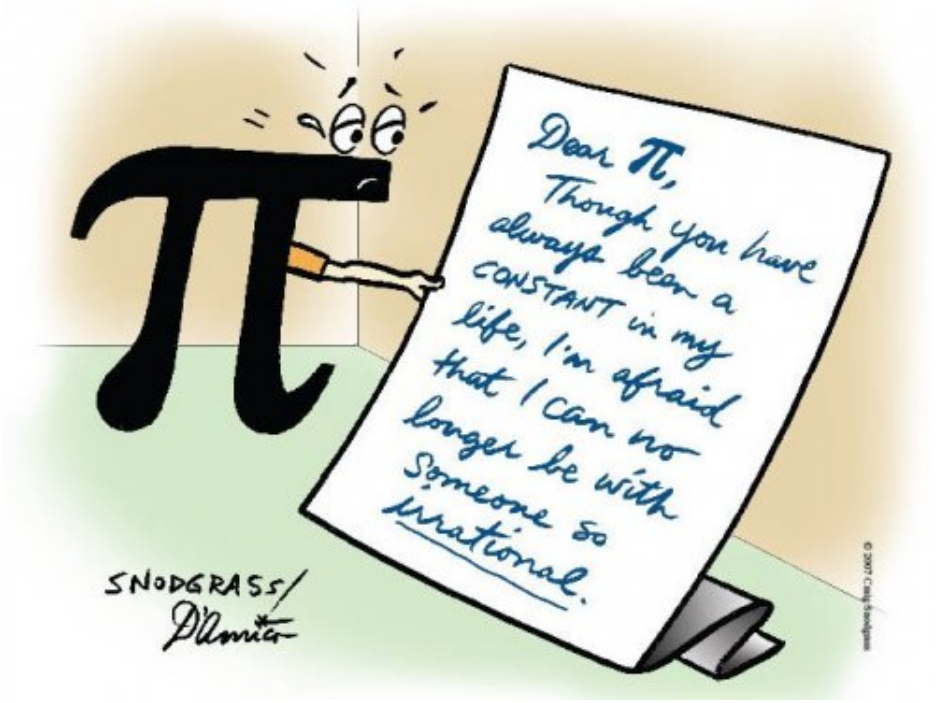

Figure 1: webcomics from Not So Humble Pi.

Class Discussion

Famous triangle points: incenter, centroid, circumcenter, orthocenter, Fermat's point. Orthocentric quadrangle.

Warm-up

Exercise 1. "I will bet you \$1" said Fred, "that if you give me \$2, I will give you \$3 in return." Tom agreed and gave Fred \$2. Who won?

Pythagorean Triples

Exercise 2. Prove that at least one number in a Pythagorean triple has to be divisible by 5.

Exercise 3. Prove that the square of the reciprocal of the height of the right triangle that corresponds to the hypotenuse equals to the sum of the squares of the reciprocals of heights corresponding to the legs.

Triangle Points

Exercise 4. The difference of two angles in a triangle is 120 degrees. Prove that the bisector from the third angle is twice longer than the altitude from the third angle.

Exercise 5. Prove that in any scalene triangle the bisector lies between the median and the height from the same angle.

Exercise 6. It is known that in a triangle the median, the bisector and the altitude from the vertex C, divide the angle into four equal parts. Find the angles of this triangle.

Exercise 7. What line should be called a median of a tetrahedron? Will all the medians of a tetrahedron meet in one point? Describe the point.

Competition Practice

Exercise 8. 2006 AIME-I. Find the least positive integer such that when its leftmost digit is deleted, the resulting integer is $1/29$ of the original integer.

Exercise 9. 2006 AIME-I. Let N be the number of consecutive 0's at the right end of the decimal representation of the product $1!2!3!4!\cdots 99!100!$. Find the remainder when N is divided by 1000.

Exercise 10. 2006 AIME-I. The number $\sqrt{104\sqrt{6}+468\sqrt{10}+144\sqrt{15}+2006}$ can be written as $a\sqrt{2}+b\sqrt{3}+c\sqrt{5}$, where a, b, and c are positive integers. Find $a \cdot b \cdot c$.