

# PIE

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<http://xkcd.com/832/>

## Class Discussion

Venn Diagrams. Principle of Inclusion and Exclusion.

## Warm-Up

**Exercise 1.** Is there 4th of July in Australia?

**Exercise 2.** Peter had 10 cows. All but 9 died. How many cows are left alive?

**Exercise 3.** Archaeologists found a coin that had a date of 48 B.C. printed on it. How valuable is the coin?

**Exercise 4.** A patient needs to get three shots with half an hour break between shots. Assuming shots themselves are instantaneous, how much time will the procedure take?

## PIE

**Exercise 5.** How many positive integers less than 100,000 are neither squares nor cubes?

**Exercise 6.** How many 9-digit numbers have the property that the product of their first and last digits is even?

**Exercise 7. Mandelbrot.** Yeechi has a deck of cards consisting of 2 through 5 of hearts and 2 through 5 of spades. She deals two cards at random to each of four players. What is the probability that no player receives a pair.

**Exercise 8.** How many positive integers less than 1000 are relatively prime to both 10 and 12?

**Exercise 9.** Tanya has 57 students each talking at least one language from the list: Russian, Spanish, Chinese. If 29 are talking Russian, 34 are talking Spanish, 33 are talking Chinese, 15 are talking Russian and Spanish, 16 are talking Spanish and Chinese, 12 are taking Russian and Chinese, then how many students are talking all three of languages?

## Competition Practice

**Exercise 10. 2002 AMC 10-A.** Points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  lie, in that order, on  $\overline{AF}$ , dividing it into five segments, each of length 1. Point  $G$  is not on line  $AF$ . Point  $H$  lies on  $\overline{GD}$ , and point  $J$  lies on  $\overline{GF}$ . The line segments  $\overline{HC}$ ,  $\overline{JE}$ , and  $\overline{AG}$  are parallel. Find  $HC/JE$ .

**Exercise 11. 2002 AMC 10-A.** A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

**Exercise 12. 2002 AMC 10-A.** The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. What is the largest integer that can be an element of this collection?

## Challenge Problems

**Exercise 13.** One day AMSA math club had a party. Every team member was at the party, but people came and left at different time. No one came to the party room twice. Every boy on the team stayed long enough to say hello to every girl. Prove that there was a moment in time when either all girls or all boys were at the party.

**Exercise 14.** Is it true that any even integer can be represented as a sum of two integers with all odd digits?

**Exercise 15. Spit Not So, Fat Fop, as if in Pan!** Write the nine words in the previous sentence on nine separate cards and have two players alternately select cards. A player wins if s/he can collect all the cards that contain a given letter. What is the strategy?