

# Translations

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## Class Discussion

M.C. Escher. Translations. Given a vector  $a$ , the slide of a figure in the direction of  $a$  at a distance equal to the length of  $a$  is called a *translation*.

**Theorem.** A translation  $F'$  of a figure  $F$  is congruent to  $F$ .

## Warm Up

**Exercise 1.** Three horses were galloping at 27 miles per hour. What was the speed of one horse?

**Exercise 2.** Every point on the plane is either black or white. Prove that there are two points of the same color at a distance of one meter.

**Exercise 3.** Can a cube of cheese three inches on a side be cut into 27 one-inch cubes with five straight cuts? What if one can move the pieces prior to cutting?

**Exercise 4.** In this problem, “truth-tellers” always tell the truth and “liars” always lie. In (1)-(3), all persons are either truth-tellers or liars.

1. There are two persons, A and B. A says, “At least one of us is a liar.” What are A and B?
2. A says, “I am a liar or B is a truth-teller.” What are A and B?
3. Now we have three persons, A, B, and C. A says, “All of us are liars.” B says, “Exactly one of us is a truth-teller.” What are A, B, and C?

4. Now we have a third type of person, called “normal,” who sometimes lies and sometimes tells the truth. A says, “I am normal.” B says, “That is true.” C says, “I am not normal.” Exactly one of A, B, C is a truth teller, one is a liar, and one is normal. What are A, B, and C?

## Translations

**Exercise 5.** There are  $n$  points on the plane. Prove that the shortest zig-zag line connecting all of them doesn't intersect itself.

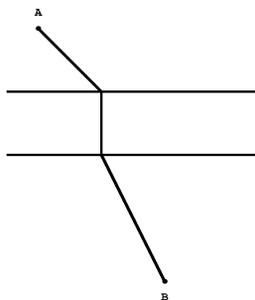
**Exercise 6.** Prove that a translation of a circle is a circle.

**Exercise 7.** Given two circles  $R$  and  $S$  and a segment  $AB$ , build a segment that is equal and parallel to  $AB$  with the end points belonging to  $R$  and  $S$ .

**Exercise 8.** Find the locus of points  $M$  the sum of whose distances from two given lines  $l_1$  and  $l_2$  is equal to a given value  $a$ .

## Challenge Problems

**Exercise 9.** Where should we build a bridge through the river defined by two parallel lines  $L_1$  and  $L_2$ , so that the road between two houses  $A$  and  $B$  on different sides of the river is minimal? (The bridge is perpendicular to lines  $L_1$  and  $L_2$ ) (see figure)



**Exercise 10.** A rectangular sheet of paper is folded so that two diagonally opposite corners come together. If the crease formed is the same length as the longer side of the sheet, what is the ratio of the longer side of the sheet to the shorter side?