# Test

Tanya Khovanova

June 1, 2015

**Your name**:
**Your grade**:

**Exercise 1. 1 point.** There were five bowls full of candy on the table. Mike ate two bowls of candy. How many bowls are there on the table now?

**Answer:**

**Exercise 2. 1 point.** Find the sum of all the coefficients of $(x^2-4x+2)^{2015}$.

**Answer:**

**Exercise 3. 1 point.** Detective Radstein is investigating a robbery. He apprehends three suspects: Anne, Bill, and Caroline. The detective knows that no one else could have participated in the robbery. During the interrogation the suspects make the following statements:

- Anne: I didn't do it. Bill did it alone.
- Bill: I didn't do it. Caroline did it.
- Caroline: I didn't do it. Bill did it.

Detective Radstein also discovered that all three suspects are members of a clan called the Halfsies. Every time they speak, they make two statements, one of which is a lie and the other of which is true. Who committed the robbery?

**Answer:**

**Exercise 4. 1 point.** What is the most probable AMC answer?

(A) $3\sqrt{2}/\pi$ (B) $\sqrt{3}/\pi$ (C) $\sqrt{3}$ (D) $6/\pi$ (E) $\sqrt{3}\pi$

**Answer:**

**Exercise 5. 1 point.** Let $SOD(n)$ be the sum of the digits of $n$. Suppose $f(n)$ is the result of iterating SOD many times until we get a single digit. That is, $f(n) = SOD(SOD(SOD \ldots (n) \ldots))$. Find $f(2^{2015})$.

**Answer:**

**Exercise 6. 1 point.** In the equation $3x^2 - 3x + 1 = 0$, what is the sum of the squares of the roots?

**Answer:**

**Exercise 7. 1 point.** What day of the week is September 10, 2015? What about May 29, 1931?

**Answer:**

**Exercise 8. 1 point.** There are four coins lying on the table in a row. Some of the coins are fake (lighter) and other coins are real. It is known that there exist at least one fake coin and one real coin. Also, every real coin lies to the left of every fake coin. Using one weighing on a balance scale find the type of every coin on the table.

**Answer:**

**Exercise 9. 1 point.** There are 16 glasses on the table, 15 of them are face up and one face down. You are allowed to change the direction of exactly four glasses at the same time. How can you turn all the glasses face up?

**Answer:**

**Exercise 10. 1 point.** Find the largest square of the form $1!+2!+3!+\ldots+n!$. Explain.

**Answer:**

**Exercise 11. 1 point.** There is an infinite wall on the plane in the form of a straight line. You have the materials to build an extra piece of wall of any shape of length $M$. For some strange reason you want to build an enclosure of the maximal area and you can use the existing piece of wall. What shape should your wall be? Why?

**Answer:**

**Exercise 12. 1 point.** There are 6 glasses on the table in a row. The first three are empty, and the last three are filled with water. How can you make it so that the empty and full glasses alternate, if you are allowed to touch only one of the glasses? (You can't push one glass with another.)

**Answer:**

**Exercise 13. 1 point.** Every next digit of number $N$ is strictly greater than the previous one. What is the sum of the digits of $9N$?

**Answer:**

**Exercise 14. 1 point.** Use induction to prove that $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.

**Answer:**

**Exercise 15. 2 points.** Solve the equation: $z^4 - 5z^3 + 6z^2 - 5z + 1 = 0$.

**Answer:**

**Exercise 16. 2 points.** One day you meet your friend Alice enjoying a nice walk with her husband Bob and their son Carl. They are excited to see you and they invite you to their party.

- Alice: Please, come to our party on Sunday at our place at 632 Elm St. in Watertown.
- Bob: My wife likes exaggerating and multiplies every number she mentions by 2.
- Carl: My dad compensates for my mom's exaggerations and divides every number he mentions by 4.
- Alice: Our son is not like us at all. He doesn't multiply or divide. He just adds 8 to every number he mentions.

Where is the party?

**Answer:**

**Exercise 17. 2 points.** Find a polynomial with integer coefficients with a root $\sqrt[3]{2+\sqrt{3}}+\sqrt[3]{2-\sqrt{3}}$.

**Answer:**

**Exercise 18. 2 points.** Describe point $M$ inside a triangle $ABC$ such that the sum of the distances from this point to the vertices of the triangle is minimal.

**Answer:**

**Exercise 19. 2 points.** A rectangular sheet of paper is folded so that two diagonally opposite corners come together. If the crease formed is the same length as the longer side of the sheet, what is the ratio of the longer side of the sheet to the shorter side?

**Answer:**

**Exercise 20. 2 points.** Prove that the second to last digit of every power of 3 is even.

**Answer:**