

Working with Square Roots

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Who is inclined towards mathematics? Take the shovels and extract roots.

Class Discussion

How to work with expressions, where p_i are polynomials in x . Cases: $p_1 + \sqrt{p_2} = p_3$, $\sqrt{p_1} + \sqrt{p_2} = p_3$, $\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3} = p_4$, $\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3} + \sqrt{p_4} = 0$. Rejoice when you have: $\sqrt{p_1 + \sqrt{p_2}} + \sqrt{p_1 - \sqrt{p_2}}$.

Warm Up

Exercise 1. Genetically engineered octopuses with an even number of arms always tell the truth, the ones with an odd number of arms always lie. Only octopuses with six, seven or eight arms are allowed to serve Neptune. The four guards from the last shift of guards at the palace have nothing better to do than count their arms: The pink one said, “Gray and I have 15 arms together.” The gray one said, “Lavender and I have 14 arms together.” The lavender one said, “Turquoise and I have 14 arms together.” The turquoise one said, “Pink and I have 15 arms together.”

What number of arms does each one have?

Exercise 2. Write a quadratic equation one root of which is the sum and the other is the product of roots of $ax^2 + bx + c = 0$.

Square Roots

Exercise 3. Factor: $x^4 + 2x^3 + 4x^2 + 3x + 2$.

Exercise 4. Simplify:

$$\left(\frac{1 + \sqrt{x}}{\sqrt{1+x}} - \frac{\sqrt{1+x}}{1 + \sqrt{x}}\right)^2 - \left(\frac{1 - \sqrt{x}}{\sqrt{1+x}} - \frac{\sqrt{1+x}}{1 - \sqrt{x}}\right)^2.$$

Exercise 5. Prove that:

$$(4 + \sqrt{15})(\sqrt{10} - \sqrt{6})\sqrt{4 - \sqrt{15}} = 2.$$

Exercise 6. Solve: $1 + \sqrt{1 + x\sqrt{x^2 - 24}} = x$.

Exercise 7. Solve: $\sqrt{3x + 4} + \sqrt{x - 4} = 2\sqrt{x}$.

Exercise 8. Solve: $\sqrt{x + \sqrt{x + 11}} + \sqrt{x - \sqrt{x + 11}} = 4$.

Exercise 9. Find $z^5 + 1/z^5$, given that $z > 0$ and $z^2 + 1/z^2 = 14$.

Exercise 10. Write a quadratic equation such that its roots are equal to the roots of $ax^2 + bx + c = 0$ plus 1.

Exercise 11. Find an equation with integer coefficients that has $\sqrt{2} + \sqrt{3}$ as its root.

Challenge Problems

Exercise 12. Find the first 1000 digits after the decimal point of $(6 + \sqrt{35})^{2015}$.

Exercise 13. Find the flaw in the following proof that $1 = -1$.

$$\sqrt{\frac{1}{-1}} = \sqrt{\frac{-1}{1}} \Rightarrow \frac{\sqrt{1}}{\sqrt{-1}} = \frac{\sqrt{-1}}{\sqrt{1}} \Rightarrow \sqrt{1}\sqrt{1} = \sqrt{-1}\sqrt{-1} \Rightarrow 1 = -1.$$

Exercise 14. Find the flaw in the following proof that $1 = -1$.

$$-1 = i^2 = \sqrt{-1}\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1.$$