Palindromic Polynomials

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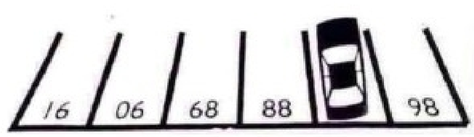
Class Discussion

A polynomial is palindromic (self-reciprocal), if the sequence of its coefficients is a palindrome. Let $P(x) = \sum_{i=0}^{n} a_i x^i$ be a polynomial of degree n, then P is palindromic iff (if and only if) $a_i = a_{n-i}$ for $i = 0 \ldots n$. For example, $(x+1)^n$ is palindromic.

Solving $ax^4 + bx^3 + cx^2 + bx + a = 0$.

Warm Up

Exercise 1. Hong Kong Elementary School First Grade Student Admission Test Question. What parking spot number is the car parked in?



Exercise 2. Smullyan. A group of friends went into an inn to have a meal. The bill amounted to 24 coins of equal value, which the men agreed to split equally. But then they discovered that two of the men had slipped away without paying their shares, so each of the remaining men had to pay one coin more. How many men were originally in the group?

Exercise 3. Smullyan. Hassan was a good friend of Ali and Ahmed. The following facts are true about them:

- Either Ali or Ahmed is the oldest of the three.
- Either Hassan is the oldest or Ali is the youngest.

Who is the oldest and who is the youngest?

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Exercise 4. Solve the equation: $1 - 27z/4 + 101z^2/8 - 27z^3/4 + z^4 = 0$.

Exercise 5. Solve the equation: $z^4 - 5z^3 + 6z^2 - 5z + 1 = 0$.

Exercise 6. Solve the equation: $(x + a)(x + 2a)(x + 3a)(x + 4a) = b^4$.

Competition Practice

Exercise 7. 2004 AMC 10. What is the value of x if $|x - 1| = |x - 2|$?

Exercise 8. 2002 AMC 10. For how many positive integers n is $n^2 - 3n + 2$ a prime number?

Exercise 9. 2002 AMC 10. Let a, b, and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. What is $a^2 - b^2 + c^2$?

Exercise 10. 2003 AMC 10. Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of $(d - 1)(e - 1)$?

Exercise 11. HMMT 2008. Determine all real numbers a such that the inequality $|x^2 + 2ax + 3a| \leq 2$ has exactly one solution in x.

Exercise 12. HMMT 2008. Positive real numbers x, y satisfy the equations $x^2 + y^2 = 1$ and $x^4 + y^4 = 17/18$. Find xy.

Exercise 13. HMMT 2008. Let $f(n)$ be the number of times you have to hit the $\sqrt{}$ key on a calculator to get a number less than 2 starting from n. For instance, $f(2) = 1$, $f(5) = 2$. For how many $1 < m < 2008$ is $f(m)$ odd?