

Geometry

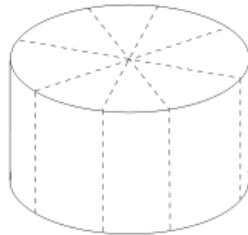
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Geometry is the art of coming to true conclusions from wrong-drawn pictures.

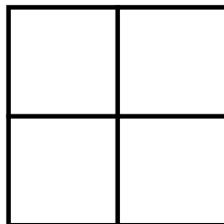
Warm Up

Exercise 1. It's easy to see that a cylinder of cheese can be cut into eight pieces with four straight cuts. What is the maximum number of pieces you



can get with three straight cuts?

Exercise 2. It is easy to divide a square into four congruent pieces (see Figure): Can you divide a square into 5 congruent pieces?



Exercise 3. A certain sheik named Hassan has eight horses. Four of them are white, three are black, and one is brown. Assuming now that Hassan's horses can talk and always tell the truth, how many of them will each say that it is the same color as another one of Hassan's horses?

Competition practice

Exercise 4. 2002 AMC 10A, Problem 18. A $3 \times 3 \times 3$ cube is formed by gluing together 27 standard cubical dice. (On a standard die, the sum of the numbers on any pair of opposite faces is 7.) What is the smallest possible sum of all the numbers showing on the surface of the $3 \times 3 \times 3$ cube?

Exercise 5. 2006 AMC 10A, Problem 19. How many non-similar triangles have angles whose degree measures are distinct positive integers in arithmetic progression?

Exercise 6. 2006 AMC 10A, Pr 23. Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent touches the circles at C and D . Lines AB and CD intersect at E , and $AE = 5$. What is CD ?

Exercise 7. 2006 AMC 10A, Problem 24. Centers of adjacent faces of a unit cube are joined to form a regular octahedron. What is the volume of this octahedron?

Challenge Problems

Exercise 8. Suppose you have a calculator that can perform arithmetic operations ($+$, $-$, $*$, $/$) and it has two special buttons: one to calculate the absolute value of a number and the other the maximum of two numbers.

1. The absolute-value-of-a-number button got broken. How can you calculate the absolute value of a number without it?
2. Now the absolute-value button got repaired but the maximum button got broken. How can you calculate the maximum of two numbers?

Exercise 9. Suppose that someone put down a copper coin, a silver coin, and a gold coin and asked you to make a statement, with the understanding that if your statement is true, then you will be given one of the three coins, but if your statement is false you will be given no coin. What statement could you make that would guarantee that you would get the gold coin?