

Common Knowledge

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October 27, 2014

“The problems for the exam will be similar to the ones discussed in class. Of course, the numbers will be different. But not all of them. Pi will still be 3.14159... ”

Class Discussion

Consecutive numbers on foreheads. Hats, when the number of red hats is less than the number of people. Professors and their published mistakes. Common knowledge.

Warm-Up

Exercise 1. Humans have 10 fingers on their hands. How many fingers are there on 10 hands?

Exercise 2. Let us call a natural number “awesome” if it can be represented as $a^b + b^a$, where a and b are natural numbers. For example, number 57 is awesome as $57 = 2^5 + 5^2$. Is 2014 awesome?

Exercise 3. Invent your own snowball sentence of at least seven words.

I do not know where family doctors acquired illegibly perplexing handwriting; nevertheless, extraordinary pharmaceutical intellectuality, counterbalancing indecipherability, transcendentalizes intercommunications' incomprehensibility.

Competition practice

Exercise 4. 2000 Streamline Olympiad. 6th-7th grade. Let A be the least integer such that the sum of all its digits is equal to 2000. Find the left-most digit of A .

Exercise 5. 1998 Streamline Olympiad. 8th-9th grade. Find three numbers such that each of them is a square of the difference of the two others.

Exercise 6. 1999 Streamline Olympiad. 9th-10th grade. The positive integers 30, 72, and N have a property that the product of any two of them is divisible by the third. What is the smallest possible value of N ?

Exercise 7. 1999 Streamline Olympiad. 9th-10th grade. You have 6 coins weighing 1, 2, 3, 4, 5 and 6 grams that look the same. The number (1, 2, 3, 4, 5, 6) on the top of each coin should correspond to its weight. How can you determine whether all the numbers are correct, using the balance scale only twice?

Exercise 8. 2000 Streamline Olympiad. 8th grade. You have six bags with coins that look the same. Each bag has an infinite number of coins and all coins in the same bag weigh the same amount. Coins in different bags weigh 1, 2, 3, 4, 5 and 6 grams exactly. There is a label (1, 2, 3, 4, 5, 6) attached to each bag that is supposed to correspond to the weight of the coins in that bag. You have only a balance scale. What is the least number of times do you need to weigh coins in order to confirm that the labels are correct?

Induction

Exercise 9. Use mathematical induction to prove that given n straight lines on a plane you can color the regions created by the lines into red and blue in such a way that any two neighboring regions are colored with different colors.

Exercise 10. Use induction to prove that if $x + 1/x$ is an integer then $x^n + 1/x^n$ is also an integer.

Exercise 11. Prove that the second to last digit of every power of 3 is even.