

# Coins 2

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## Class Discussion

Discuss problems 3 and 8 about privacy of coins.

## Warm-up

**Exercise 1.** Ten crows were sitting on a fence. A farmer shot one. How many were left?

**Exercise 2.** John was born on February 29, 1896. How many birthdays has he celebrated properly by the age of 33?

**Exercise 3.** You present 100 identical coins to a judge, who knows that among them are either one or two fake coins. All the real coins weigh the same and all the fake coins weigh the same, but the fake coins are lighter than the real ones.

You yourself know that there are exactly two fake coins and you know which ones they are. Can you use a balance scale to convince the judge that there are exactly two fake coins without revealing any information about any particular coin?

**Exercise 4.** A certain hobo who is skilled at making cigarettes can turn any 4 cigarette butts into a single cigarette. Today, this hobo has found 24 cigarette butts on the street. Assuming he smokes every cigarette he can, how many cigarettes will he smoke today?

## Competition Practice

**Exercise 5. Moscow Olympiad 1936.** Find a four-digit square such that the first two digits are the same and the last two digits are the same.

**Exercise 6. Moscow Olympiad 1936.** The sides of a rectangle and its diagonal are integers. Prove that the area of the rectangle is divisible by 12.

**Exercise 7. 2015 Moscow Olympiad. 7th grade.** Is it possible to put 2015 natural numbers on a circle so that the ratio of any two neighbors is a prime number?

## Challenge Problems

**Exercise 8.** You present 100 identical coins to a judge, who knows that among them are either two or three fake coins. All the real coins weigh the same and all the fake coins weigh the same, but the fake coins are lighter than the real ones.

You yourself know that there are exactly three fake coins and you know which ones they are. Can you use a balance scale to convince the judge that there are exactly three fake coins without revealing any information about any particular coin?

**Exercise 9.** I am in a 100-story building. I have with me two glass balls. I know that if I throw the ball out of the window, it won't ever break if the floor number is less than  $X$ , and it will always break if the floor number is equal to or greater than  $X$ . Assuming that I can reuse the balls which don't break, find  $X$  in the minimum number of throws.

**Exercise 10.** I have  $N$  coins, one of them is fake and is lighter than the others. I also have a balance with two cups. I can put some number of coins into each cup, and the balance shows me which set of coins is lighter. Using the balance the fewest number of times, find the fake coin. Unlike in the previous homework this time you have to say what all of your weighings will be before you actually do them.

**Exercise 11.** Suppose you mark the time when the sun is at its highest on one day and on the next. What is the difference: exactly one day, more than one day or less than one day? What if you chose a bright star, say Sirius, instead of the sun? Does the answer change? If so, how?