Test

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Your name:

Your grade:

Exercise 1. 1 point. A family photo contained: one grandfather, one grandmother, two fathers, two mothers, six children, four grandchildren, two brothers, two sisters, three sons, three daughters, one father-in-law, one mother-in-law, one daughter-in-law.

29 people you may think, but no! What is the fewest number of people that could have been in the photo?

Answer:

Exercise 2. 1 point. Half of zero is still zero. What other number can be halved to make zero?

Answer:

Exercise 3. 1 point. A ship is docked in the harbour. Over the side hangs a rope ladder with rungs a foot apart. The tide rises at a rate of 9 inches per hour. At the end of six hours, how much of the rope ladder will still remain above water, assuming that 9 feet were above the water when the tide began to rise?

Answer:

Exercise 4. 1 point. How can you make the following equation correct without changing it at all? $8 + 8 = 91$.

Answer:

Exercise 5. 1 point. At noon, you look at the clock in your bedroom. The big hand is on the five and the little hand is in between the 3 and the 4. What time is it?

Answer:

Exercise 6. 1 point. There are 2 hourglasses measuring 7 and 4 minutes respectively. How do you measure 5 minutes? Explain.

Answer:

Exercise 7. 1 point. How many numbers between 1 and 1000 are not divisible by 3 or 7?

Answer:

Exercise 8. 1 point. How many 5-digit numbers are there with at least one odd digit?

Answer:

Exercise 9. 2 point. A faulty car odometer proceeds from digit 4 to digit 6, always skipping the digit 5, regardless of position. For example, after traveling one mile the odometer changed from 000049 to 000060. If the odometer now reads 002917, how many miles has the car actually traveled?

Answer:

Exercise 10. 2 points. Count the number of subsets of $\{1, 2, \ldots, 10\}$ that contain no consecutive integers. Explain why.

Answer:

Exercise 11. 2 points. The 100 game: two players start from 0 and alternatively add a number from 1 to 10 to the sum. The player who reaches 100 wins. List all P-positions.

Answer:

Exercise 12. 2 points. There are two people, A and B, each whom is either a knight or a knave. A makes the following statement: "At least one of us is a knave." What are A and B?

Answer:

Exercise 13. 2 points. Is number $21^{10} - 1$ divisible by 2200? Explain.

Answer:

Exercise 14. 2 points. Tanya decided to buy balloons for her math party. There are 4 colors of balloons at the Star Market and Tanya needs 6 balloons. In how many ways can Tanya buy her balloons?

Answer:

Exercise 15. 4 points. A group of five friends decide to exchange gifts as secret Santas. Each person writes their name on a piece of paper and puts it in a hat and then each person randomly draws a name from the hat to determine who has them as their secret Santa.

What is the probability that at least one person draws their own name?

Answer: