# The Simpson's Paradox

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The average person is mean.

## Class Discussion

The Simpson's paradox. The AIDS puzzle.

## Competition practice

**Exercise 1. AMC.** Keiko tosses one penny and Ephraim tosses two pennies. What is the probability that Ephraim gets the same number of heads that Keiko gets?

**Exercise 2. 2002 AMC 10B Problem 9.** Using the letters A, M, O, S, and U, we can form 120 five-letter "words". If these "words" are arranged in alphabetical order, then what position does the "word" USAMO occupy?

**Exercise 3. 2002 AMC 10B Problem 25.** When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

**Exercise 4. 2003 AMC 10A Problem 15.** What is the probability that a randomly picked integer in the set $\{1, 2, 3, \ldots, 100\}$ is divisible by 2 and not divisible by 3?

## Challenge Problems

**Exercise 5.** A group of five friends decide to exchange gifts as secret Santas. Each person writes their name on a piece of paper and puts it in a hat and then each person randomly draws a name from the hat to determine who has them as their secret Santa.

What is the probability that at least one person draws their own name?

**Exercise 6.** Tanya flips a coin 10 times and gets 10 tails in a row. What is the probability that the next flip is a tail?

**Exercise 7.** In how many ways can you run up a staircase with $n$ steps, taking steps of odd sizes where the order is relevant and there is no other restriction on the number or the size of each step taken?

**Exercise 8. Of monkeys and children**. Apinayé belongs to the Ge language family of Brazil. Currently it is spoken by less than 800 people, and therefore is seriously endangered. The following are some sentences in Apinayé, along with their English translations. You will see some letters here that do not occur in the English or Portuguese writing systems. You do not need to know exactly how these letters are pronounced in order to solve this problem:

1. Kukrẽ kokoi. — "The monkey eats."
2. Ape kra. — "The child works."
3. Ape kokoi ratš. — "The big monkey works."
4. Ape mï mɛtš. — "The good man works."
5. Ape mɛtš kra. — "The child works well."
6. Ape punui mï piηetš. — "The old man works badly."

A1 (practical). Translate the following into English:

1. Ape ratš mï mɛtš.
2. Kukrẽ ratš kokoi punui.
3. Ape piηetš mï.

A2 (practical). Translate the following into Apinayé:

1. "The big child works a long time."
2. "The old monkey eats a lot."

A3 (theoretical). Explain the meanings of the following words: ratš: mɛtš: piηetš: