Logic Formulae

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Student: Teacher, would you punish me for something I didn't do?
Teacher: No, why?
Student: I didn't do my homework.

Class Discussion

Logic operations: NOT $\neg$, AND $\wedge$, OR $\vee$, IF $\rightarrow$. Logic formulae: $\neg(p \wedge q) = \neg p \vee \neg q$, $\neg(p \vee q) = \neg p \wedge \neg q$. Disjunctive normal form.

Warm-Up

Exercise 1. Suspects A, B, and C are being questioned. The following facts are established: 1) Only A, B or C could have committed the robbery. 2) A never commits a crime alone. 3) C is innocent. Is B guilty?

Exercise 2. A, B, and C are murder suspects. The following facts are established: 1) If A is not guilty or B is guilty, then C is guilty. 2) If A is not guilty, then C is not guilty. Is A guilty?

Logic

Exercise 3. Are the following statements true? 1) Elephants can fly and $2+2=4$. 2) People have three legs or the Earth is roundish. 3) If 4 is prime, then $2+2=5$. 4) The Earth is cubic if and only if fish can speak French.

Exercise 4. Denote the statement "John loves Mary" by p, and "Mary loves John by q. Write in a symbolic form the following statements: 1) Mary and John love each other. 2) It is not true that Mary and John do not love each other. 3) Mary is loved by John, but she doesn't reciprocate.

Exercise 5. "John has more than a thousand books," said Pete. "No, he has less than one thousand books," said Ann. "He, surely, has at least one book," said Mary.

If only one statement is true, how many books John has?

Exercise 6. Build the truth tables for the following three statements: 1) $\neg p \to \neg q$, 2) $(p \wedge q) \to q$, 3) $((p \to \neg q) \wedge (q \leftrightarrow \neg p)) \to \neg(\neg p \vee q)$.

Exercise 7. Simplify the following statements: 1) $p \to (q \to p)$, 2) $((p \to q) \to q) \to (\neg(p \to q) \vee q)$, 3) $(p \wedge q) \wedge \neg(p \vee q)$.

Exercise 8. Folks living in Trueton always tell the truth. Those who live in Lieberg, always lie. People living in Alterborough alternate strictly between truth and lie. One night a call came into 911: "Fire, help!" The operator couldn't ID the phone number, so he asked, "Where are you calling from?" "Lieberg."

Assuming no one had overnight guests from another town. Should the firemen hurry? If so, then where to?

Exercise 9. Assume the following is true: A sane person always takes an umbrella to fly a balloon. If you take an umbrella with you, you are guaranteed a sunny day. It is raining and I am flying in the balloon.

Can you conclude that I am insane?

Challenge Problems

Exercise 10. There are two guards and two doors, each guard guarding a door. One door leads to freedom, the other to death. One guard always lies, the other always tells the truth. They know where the two doors go and who is lying and who is not. You do not know which guard is which. You may ask one yes or no question. What do you ask to determine which door goes to freedom?

Exercise 11. Consider a logic system such that there are three possibilities for every statement: true, false and maybe. Does the following formula hold in this system: $\neg(\neg p) = p$. Why?

Exercise 12. A criminal is sentenced to death. He is offered the last word. He is allowed to make one statement. If the statement is true, the criminal will be put to the electric chair. It the statement is false, he will be hanged. Can you find a good piece of advice for this man?