Divisibility

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Class Discussion

Guessing a color trick. Divisibility by numbers. The significance of 1001. The flip-flop game.

Warm-Up

Exercise 1. Four adventurers (Alex, Brook, Chris and Dusty) need to cross a river in a small canoe.

The canoe can only carry up to 100kg.

Alex weighs 90kg, Brook weighs 80kg, Chris weighs 60kg and Dusty weighs 40 kg, and they have 20kg of supplies.

How do they get across?

Exercise 2. There are three people (Alex, Brook and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth.

Alex says: "Cody is a knave." Brook says: "Alex is a knight." Cody says: "I am the spy."

Who is the knight, who the knave, and who the spy?

Divisibility

Exercise 3. Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.

Exercise 4. Prove that $7^{2n} - 5^{2n}$ is divisible by 24.

Exercise 5. For what primes p does there exist a number consisting only of ones that is divisible by p.

Exercise 6. Prove that for any n the number written with 3^n ones is divisible by 3^n.

Exercise 7. A three-digit prime number has all distinct digits. What is the number's last digit, if it is equal to the sum of the first two digits?

Exercise 8. The sum of the digits of a number is 2013. Can it be square?

Challenge Problems

Exercise 9. I have three special four-sided dice. They have one letter on each side. When I roll them together I get three random letters which I try to rearrange into a word. In my eight goes so far I have made the words:

CAT, SON, POD, RIG, PEG, TAP, DIN, APE

What are the letters on each dice?

Exercise 10. During a recent census, a man told the census taker that he had three children. When asked their ages, he replied, "The product of their ages is 36. The sum of their ages is the same as my house number."

The census taker ran to the door and looked at the house number. "I still can't tell," she complained. The man replied, "Oh that's right, I forgot to tell you that the oldest one likes chocolate pudding."

The census taker promptly wrote down the ages of the 3 children. How old are they?