Planar Graphs

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Class Discussion

Conway's nose trick. Review of Euler walks. Planar graphs. A proof that K_5 is not planar.

Warm-Up

Exercise 1. Five women have lunch together seated around a circular table. Ms Osborne is sitting between Ms Lewis and Ms Martin. Ellen is sitting between Cathy and Ms Norris. Ms Lewis is between Ellen and Alice. Cathy and Doris are sisters. Betty is seated with Ms Parker on her left and Ms Martin on her right. Match the first names with the surnames.

Exercise 2. One night, a man receives a call from the police. The police tell the man that his wife was murdered, and that he should reach the crime scene as soon as possible. The man drops the phone, shocked, and drives 20 minutes to the crime scene. As soon as he reaches the crime scene, the police arrest him, and he is convicted of murder. How did the police know that he committed the crime?

Exercise 3. The year 1978 has an unusual property. When you add the 19 to the 78, the total is the same as the middle two digits (97). What will be the next year to have this same property?

Exercise 4. A man was found dead in his study. He was slumped over his desk and a gun was in his hand. There was a cassette recorder on his desk. When the police entered the room and pressed the play button on the tape recorder they heard: "I can't go on. I have nothing to live for." Then there was the sound of a gunshot. How did the detective immediately know that the man had been murdered and it wasn't a suicide?

Graphs

Exercise 5. Prove that a graph can't have exactly one vertex with an odd degree.

Exercise 6. Prove that if a connected graph has no more than two vertices with an odd degree, then there exists an Euler walk on that graph. That is, you can draw this graph without following each edge twice.

Exercise 7. What is the maximum number of edges that a planar graph with 4 vertices can have? What about 5 or 6 vertices? Any conjectures?

Competition Practice

Exercise 8. PUMaC 2012. If x, y, and z are real numbers with $\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} = 36$, find $2012 + \frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y}$.

Exercise 9. PUMaC 2012. How many ways can 2^{2012} be expressed as the sum of four (not necessarily distinct) positive squares?

Challenge Problems

Exercise 10. Suppose there are three cottages on a plane and each needs to be connected to the gas, water, and electric companies. Using a third dimension or sending any of the connections through another company or cottage is disallowed. Make all nine connections without any of the lines crossing each other.