Extreme Principle

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Class Discussion

Extreme Principle.

Problem. Let B and W be finite sets of black and white points, respectively, in the plane, with the property that every line segment that joins two points of the same color contains a point of the other color. Prove that both sets must lie on a single line segment.

Solution. Consider the triangle of the smallest area.

Warm-Up

Exercise 1. A store sells letter magnets. The same letters cost the same. Different letters might not cost the same. The word ONE costs \$6, the word TWO costs \$9, and the word ELEVEN costs \$16. What is the cost of TWELVE?

Exercise 2. There are numbers 1, 2, ..., 100 on the board. In one step you are allowed to replace any two of them with their sum or product. What is the largest number you can get at the end?

Exercise 3. Athos, Porthos, and Aramis were rewarded with six coins: three gold and three silver. Each got two coins. Atos doesn't know what kind of coins others got, but knows his own coins. Ask him one question such that he can answer "Yes," "No," or "I do not know." and you will be able to figure out his coins.

Exercise 4. A plane started from Boston. It flew 1000 miles North, then 1000 miles West, then 1000 miles South, then 1000 miles East. Is it back in Boston? If not, which way from Boston it is?

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Exercise 5. Snow White bought gifts for the seven dwarfs. She bought 7 drums of different sizes and 7 pairs of drums stick, also in different sizes. The dwarfs who saw that both their drums and drums sticks are bigger than someone else's started to drum. What is the largest number of dwarfs that can be drumming?

Exercise 6. There are different coins lying on the table. No coin lies on top of another. Prove that you can move out one of the coins.

Exercise 7. In a volleyball tournament all teams played with each other once. It is known that 20 percent of all teams lost all games. How many teams are there?

Exercise 8. There are 15 planets such that the distances between any two of them are distinct. There is one astronomer on each planet who studies the planet closest to his planet. Prove that there is a planet no astronomer studies.

Exercise 9. Prove that a convex polyhedron has two faces with the same number of edges.

Competition Practice

Exercise 10. HMNT 2005. Guts Round. Let $m \circ n = (m+n)/(mn+4)$. Compute $((\cdots((2005 \circ 2004) \circ 2003) \circ \cdots \circ 1) \circ 0)$.