Sum of Divisors

Tanya Khovanova

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Question: Why did the chicken cross the Moebius strip?
Answer: To get to the other ... er, um ...

Class Discussion

Perfect numbers. Abundant and deficient numbers.

Denote $\sigma(n)$ the sum of divisors of n. If $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ is a prime decomposition of n, then the sum of divisors of n is:

$$\sigma(n) = \frac{p_1^{a_1+1} - 1}{p_1 - 1} \cdots \frac{p_k^{a_k+1} - 1}{p_k - 1}.$$

Warm-Up

Exercise 1. Two friends played chess for four hours. How many hours have each of them played chess for?

Exercise 2. In the equality $101 - 102 = 1$ move one of the digits so that it becomes correct.

Sum of Divisors

Exercise 3. For every $1 \leq k \leq 6$, find the smallest natural number with k distinct divisors.

Exercise 4. Does there exist a positive integer such that the product of its divisors ends in exactly 2011 zeroes?

Exercise 5. HMMT 2006, Guts round. Compute the positive integer less than 1000 which has exactly 29 positive proper divisors. (Here we refer to positive integer divisors other than the number itself.)

Exercise 6. Find a positive integer n, such that $\tau(n) = 6$, $\sigma(n) = 28$.

Competition Practice

Exercise 7. HMMT 2005, Guts round. For how many integers n between 1 and 2005, inclusive, is $2 \times 6 \times 10 \cdots (4n - 2)$ divisible by $n!$?

Exercise 8. HMMT 2007, Guts round, 6 points. Compute the largest positive integer n such that $\frac{2007!}{2007^n}$ is an integer.

Exercise 9. AMC 12 2005. Two distinct numbers a and b are chosen randomly from the set $\{2,\ 2^2,\ 2^3,\ \ldots,\ 2^{25}\}$. What is the probability that $\log_a b$ is an integer?

Challenge Problems

Exercise 10. Moscow Olympiad 2011. Stan and Kyle each have three sticks of total length 1 meter. Both can make a triangle out of their sticks. At night Eric secretly switched one of Stan's sticks and one of Kyle's sticks. First thing in the morning, Stan tried to make a triangle out of his sticks and couldn't. Can Kyle make a triangle out of his sticks?