Solving a Triangle

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Class Discussion

Solving a triangle. Sides a, b, c. Opposite angles A, B, C.

- The angles sum up to 180°.
- Law of Sines:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
- Law of Cosines:
$$c^2 = a^2 + b^2 - 2ab\cos(C).$$

Solving a triangle if you know three measurements out of six:

- AAA — three angles. The triangle is defined only up to scaling.
- AAS — two angles and one side, not in between.
- ASA — two angles and one side in between.
- SAS — two sides and an angle in between.
- SSA — two sides and an angle not in between (there could be two different solutions)
- SSS — three sides.

Warm-Up

Exercise 1. A large truck is crossing a bridge 1 mile long. The bridge can only hold 14000 lbs, which is the exact weight of the truck. The truck makes it half way across the bridge and stops. A bird lands on the truck. Will the bridge collapse? Explain.

Exercise 2. How many eggs can you put in an empty basket?

Exercise 3. How do you make seven even?

Solving Triangles

Exercise 4. Solve the triangle: two angles are $45°$ and $30°$ and the side between is 10.

Exercise 5. Solve the triangle: two sides are 10 and the angle between them is $45°$.

Exercise 6. Make a complete analysis of every case. Find out when there are no solutions, and there is a unique solution for each case: AAS, ASA, SAS, SSA, SSS.

Competition Practice

Exercise 7. AMC 10, 2002. Points A, B, C, and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.

Exercise 8. AMC 10, 2002. A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

Exercise 9. 2007 Irish Mathematical Olympiad. Prove that a triangle ABC is right-angled if and only if $\sin^2 A + \sin^2 B + \sin^2 C = 2$.

Knights and Knaves

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

Exercise 10. You meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves."

Can you determine who is a knight and who is a knave?

Exercise 11. You meet two inhabitants: Peggy and Zippy. Peggy tells you that "of Zippy and I, exactly one is a knight". Zippy tells you that only a knave would say that Peggy is a knave.

Can you determine who is a knight and who is a knave?