

Simpson's Paradox

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Class Discussion

Simpson's paradox.

Warm-Up

Exercise 1. The total of 12 children are riding in a school bus to school. Assuming there are no disabled people on the bus, how many legs are in the bus?

Exercise 2. May 5 of 1955 can be written as 5/5/55. How many times during the 20th century the date in the format month day and the last two digits of the year can be written with the same digit?

Exercise 3. A delivery guy in a big hotel had to work during the power outage. On Monday he had to make five trips to the tenth floor. On Tuesday he had to make ten trips to the fifth floor. On which day he had to walk more stairs?

Exercise 4. How many integers have all their digits in increasing order?

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Exercise 5. Change the data in the table below, so that the total percentages of admitted students is 50-50, suggesting that there is no bias. Make the data show that there is gender bias.

Two departments: Math—many applicants accepted
 English—few applicants accepted

| | Math | | English | | Total | | |
|---------|-------|--------|---------|--------|-------|--------|---------|
| Males | 90 | 10 | 1 | 9 | 91 | 19 | 83% |
| | admit | denied | admit | denied | admit | denied | % admit |
| Females | 9 | 1 | 10 | 90 | 19 | 91 | 17% |

Exercise 6. A community has two hospitals. Hospital *A* is a large medical center, while Hospital *B* is a more fashionable and much more expensive hospital where most patients are wealthy. An article in the local paper claims that a higher percentage of surgery patients die at Hospital *A* than at Hospital *B*, and deplores the fact that people who are less well off are disadvantaged. It also recommends to the people in the community that if they can afford it, they should choose to have their surgery in Hospital *B* rather than *A*.

A more detailed look at the number of surgery patients in the last few months at both Hospitals, taking into account also whether the incoming patients were in good or poor health, shows the following:

| | HOSPITAL A | | HOSPITAL B | |
|----------|-------------|-------------|-------------|-------------|
| | Good Health | Poor Health | Good Health | Poor Health |
| Died | 4 | 57 | 5 | 8 |
| Survived | 559 | 1422 | 585 | 196 |

1. What are the percentages of patients admitted for surgery who are in bad health prior to the operation? (in *A* and *B*):
2. What are the total percentages of patients who died? (in *A* and *B*):
3. What are the percentages of patients in previously good health who died? (in *A* and *B*):
4. What are the percentages of patients in previously poor health who died? (in *A* and *B*):
5. Describe this paradox in your own words.

Exercise 7. Low birth-weight children born to smoking mothers have a lower infant mortality rate than the low birth weight children of non-smokers. The same is true of children born to poor parents, and of children born at high altitude; these are all examples of Simpson's paradox. Explain why mother's shouldn't smoke to decrease the mortality rate.

Competition Practice

Exercise 8. Purple Comet, 2005. Find the number of nonnegative integers n for which $(n^2 - 3n + 1)^2 + 1$ is a prime number.

Challenge Problems

Exercise 9. Three points D_1 , D_2 , and D_3 are three vertices of three isosceles trapezoids. It is known that three other vertices (A , B , and C) are the same for all three trapezoids. Build the trapezoids.

Exercise 10. I think of two integers from 1 to 9, not necessarily distinct. I tell Alice their product and Bob their sum.

1. Alice: "I don't know the numbers."
2. Bob: "I don't know the numbers."
3. Alice: "I don't know the numbers."
4. Bob: "I don't know the numbers."
5. Alice: "I don't know the numbers."
6. Bob: "I don't know the numbers."
7. Alice: "I don't know the numbers."
8. Bob: "I don't know the numbers."
9. Alice: "Now I know the numbers."

Assuming that Alice and Bob are infinitely intelligent, what are the numbers I thought of?