

Hard Problems from Competitions

Tanya Khovanova

January 23, 2012

Class Discussion

Registration for NACLO is open.

Warm-Up

Exercise 1. Peter Piper picked a peck of pickled peppers. Can you spell that without using any P?

Exercise 2. Why do you always find what you are looking for in the last place you look?

Exercise 3. On an island every person either always tells the truth, lies, or alternates true or false answers. You meet an islander Mike. How, in two yes-no questions, can you determine whether Mike is a truth-teller, a liar, or an alternate?

Problem 25

Exercise 4. AMC 12, 2000, Problem 25. Eight congruent equilateral triangles, each of a different color, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two colored octahedrons are distinguishable if neither can be rotated to look just like the other.)

Exercise 5. Do the previous problem for the other four Platonic solids: a tetrahedron, a cube, a dodecahedron, an icosahedron.

Exercise 6. AMC 10 B, 2003, Problem 25. How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

Exercise 7. MAML, 2011, Problem 25. Find the sum of all real solutions of: $\sqrt{|x+4|} + \sqrt{|x-1|} = \sqrt{|x-4|}$.

Exercise 8. MAML, 2010, Problem 25. Consider an arithmetic sequence of integers with the property that $a_{n+1} = a_n - 6$. The sum $a_1 + a_2 + a_3 + a_4 + a_5$ equals k^2 where k is a positive integer. If $k > a_5$, what is the largest possible value of a_2 ?

Challenge Problems

Exercise 9. The formulae for chemical molecules and their names are given below in mixed order:

C3H8, C4H6, C3H4, C4H8, C7H14, C2H2;
Heptene, Butyne, Propane, Butene, Ethyne, Propyne.

1. Match the formulae with their names. Explain your solution.
2. Write the names of the molecules with the following formulae: C2H4, C2H6, C7H12.
3. Write the formulae for the following molecules: Propene, Butane.

Exercise 10. The following Guaraní verb forms are listed along with their English translations.

1. japyhyta — We will be catching
2. nohyvykõiri — He is not enjoying
3. ombokapu — He is shooting
4. pemomaitei — You are greeting
5. ndokarumo'ãi — He will not be eating
6. ndapevo'oima — You were not taking
7. napekororõmo'ãi — You will not be crying
8. noñe'ëi — He is not talking
9. okororõ — He is crying
10. ndajajupirima — We were not waking up
11. ahyvykõima — I was enjoying
12. añe'ëta — I will be talking

13. namomaiteiri — I am not greeting
14. japurahei — We are singing

Translate into English:

1. akaruma
2. ojupita
3. ndavo'omo'ãi
4. napekororõ
5. ndapyhyima

Translate into Guaraní:

1. you are not shooting
2. he is not singing
3. we will be eating
4. I will not be singing

Notes: “you” is always plural in the sentences above. A squiggle over a vowel indicates that it is nasal (pronounced partly through the nose). The letter ñ is pronounced like the sound in the middle of “piñata” or “onion”. The letter y is pronounced like the “u” in “cut”. The letter j and the apostrophe (') are specific consonants. Guaraní is one of the official languages (along with Spanish) of Paraguay, where it is spoken by 94% of the population.