Interesting Angles

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Any curve going around your boss is shorter than a straight line passing him.

Class Discussion

We only need to discuss angles between $0°$ and $90°$. Well known angles: $90°$, $60°$, $30°$, $45°$.

Sums and differences:

- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double and half angles:

- $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
- $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

Interesting angles: $72°$, $36°$ and $18°$.

Warm-Up. Physics

Exercise 1. What is expected to provide more adrenalin: skating on thin ice or cross-country skiing on the same thin ice?

Exercise 2. Why balloons are spherical and not in the shape of cubes?

Exercise 3. You bought a cup of coffee. You are about to add milk when you get a phone call. You do not like drinking coffee while you talk on the phone. Should you add milk to coffee before answering the phone or after, if you like your coffee hot?

Interesting angles

Exercise 4. Find $\sin 75°$, $\sin 18°$, $\sin 15°$, $\sin 22.5°$, $\sin 3°$.

Exercise 5. Find all triangles such that their angles form an arithmetic progression and their sides: a) an arithmetic progression and b) geometric progression.

Exercise 6. Prove that $\sin 10°$ is irrational.

Exercise 7. Triangle ABC is such that $\sin A + \cos B = \sqrt{2}$ and $\cos A + \sin B = \sqrt{2}$. Find C.

Competition Practice

Exercise 8. You have more than 3 points on a plane. Prove that you can divide them into two groups so that no line can separate the groups.

Challenge problem

Exercise 9. Five people, A, B, C, D, E are related to each other. Four of them make one true statement each as follows:

1. B is my father's brother.
2. E is my mother-in-law.
3. C is my son-in-law's brother.
4. A is my brother's wife.

Each person who is mentioned is one of the five (e.g. when someone says 'B is my father's brother' you can be sure that 'my father' as well as 'my father's brother' is one of A, B, C, D, E). Find out who made which of the four statements and how the five people are related.

Exercise 10. The distance between the towns A and B is 1000 miles. There is 3000 apples in A, and the apples have to be delivered to B. The available car can take 1000 apples at most. The car driver has developed an addiction to apples: when he has apples aboard he eats 1 apple with each mile made. Figure out the strategy that yields the largest amount of apples to be delivered to B.