Pythagorean Triples

Tanya Khovanova

February 14, 2011

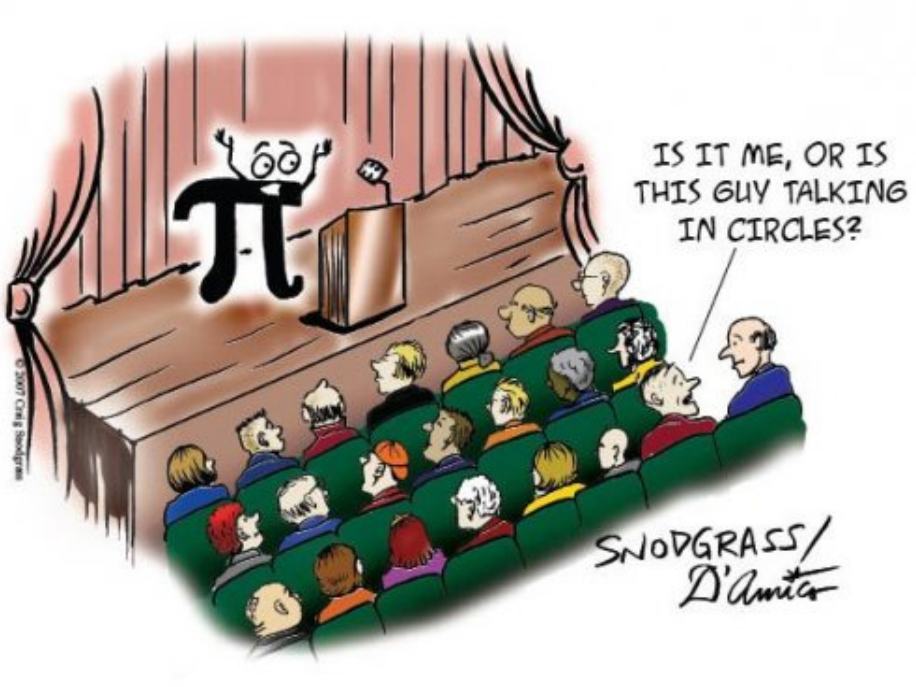

Figure 1: webcomics from Not So Humble Pi.

Class Discussion

For coprime integers m and n of different parity, $n^2 - m^2$, $2mn$, $m^2 + n^2$ form a primitive Pythagorean triple. A proof that a primitive Pythagorean triangle has an odd and an even leg.

Warm-up

Exercise 1. Is it possible that two triangles are not congruent, but all the angles of the first triangle are equal to the corresponding angles of the second triangle and two sides of the first triangle are congruent to two sides of the second triangle.

Exercise 2. Find all rectangles with integer sides so that their perimeter equals their area.

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Exercise 3. Find all Pythagorean triangles such that one leg is shorter than the hypotenuse by 1.

Exercise 4. In the right triangle the in-circle touches the hypotenuse at the point that splits the hypotenuse at segments of length 5 and 12. Find the legs of the triangle.

Exercise 5. Two vertices of a square lie on a circle of radius R, the other two — on a tangent to the circle. Find the diagonal of the square.

Exercise 6. The hypotenuse of a right triangle is a side of a square that lies on the other side of the triangle. Find the distance from the triangle vertex that corresponds to the right angle to the center of the square. The legs of the triangle are a and b.

Competition Practice

Exercise 7. 2006 AIME-I. In quadrilateral $ABCD$, $\angle B$ is a right angle, diagonal AC is perpendicular to CD, $AB = 18$, $BC = 21$, and $CD = 14$. Find the perimeter of $ABCD$.

Exercise 8. 2006 AIME-I. Let set A be a 90-element subset of $\{1,\ 2,\ 3,\ \ldots,\ 100\}$, and let S be the sum of the elements of A. Find the number of possible values of S.