Pigeonhole Principle

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Class Discussion

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Exercise 1. Can you put 44 pebbles into 9 piles so that each pile has a different number of pebbles?

Exercise 2. Humans can't have more than 400,000 hairs on their heads. Prove that there are at least three people in Manhattan that have the same number of hairs on their heads. The population of Manhattan is about 1,630,000.

Exercise 3. Is that true that given 6 integers you can always find two so that the difference is divisible by 5? What about the sum?

Exercise 4. Is it true that there exist two powers of two such that their difference is divisible by 2010?

Exercise 5. 101 hats are colored into 11 colors. Prove that there exist 11 hats of all different colors or that there exist 11 hats or the same color.

Exercise 6. Prove that given seven integers you can find three such that their sum is divisible by 3.

Exercise 7. Prove that for any number N you can find a number that is written only with zeroes and ones that is divisible by N.

Competition Practice

Exercise 8. 2001 AMC 10. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

Exercise 9. 2001 AMC 10. In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to AD, with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $AB \cdot CD$?

Exercise 10. 2001 AMC 10. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

Exercise 11. Moscow Linguistics Olympiad. You are given sentences in Niuean language with their translations into English:

1. To lele e manu. — The bird will fly.
2. Kua fano e tama. — The boy is walking.
3. Kua koukou a koe. — You are swimming.
4. Kua fano a ia. — He is walking.
5. Ne kitia he tama a Sione. — The boy saw John.
6. Kua kitia e koe a Pule. — You are seeing Pule.
7. To kitia e Sione a ia. — John will see him.
8. Ne liti e ia e kulï. — He left the dog.
9. Kua kai he kulï e manu. — The dog is eating the bird.

Translate the following sentences into Niuean: John swam. You will eat the dog. Pule is leaving you. The bird will see the boy. The dog is flying.

Challenge Problems

Exercise 12. A king decides to give 100 of his prisoners a test. If they pass, they can go free. Otherwise, the king will execute all of them. The test goes as follows: The prisoners stand in a line, all facing forward. The king puts a hat of one of seven colors (red, orange, yellow, green, blue, indigo, violet) on each prisoner. The prisoners can only see the colors of the hats in front of them. Then, in any order they want, each one guesses the color of the hat on their head. Other than that, the prisoners cannot speak. To pass, no more than one of them may guess incorrectly. If they can agree on their strategy beforehand, how can they be assured that they will survive?