

Algebra Review

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Class Discussion

Dividing treasure between two people.

Warm-Up

These three problems are from the book *The Riddle of Scheherazade* by Raymond Smullyan. In each of them a robbery happened and there were three suspects: Abu, Ibn and Hasib. Unless stated otherwise, exactly one suspect was guilty.

Exercise 1. A valuable sword was stolen. Ibn claimed that Hasib stole it, and Hasib claimed that Abu stole it. Now it wasn't certain that any of these three stole the sword, but it was later determined that no one innocent lied. Also, the sword was stolen by only one person. Can it be determined who stole the sword?

Exercise 2. This time a valuable clock was stolen. Abu claimed that Hasib was innocent, and Hasib claimed that Ibn was innocent. Ibn's statement was not recorded. Curiously enough, the guilty person told the truth and the two innocent ones both lied. Who stole the clock?

Algebra Review

Exercise 3. Solve the equation: $\frac{1}{x(x+2)} - \frac{1}{(x+1)^2} = \frac{1}{12}$.

Exercise 4. Simplify: $\sqrt[4]{6x(5 + 2\sqrt{6})} \cdot \sqrt{3\sqrt{2x} - 2\sqrt{3x}}$.

Exercise 5. Solve the equation $(x - 2)^6 + (x - 4)^6 = 64$.

Competition Practice

Exercise 6. 2000 AMC10. Let A , M , and C be nonnegative integers such that $A + M + C = 10$. What is the maximum value of $A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$?

Exercise 7. AMC12. Let $P(x) = (x - 1)(x - 2)(x - 3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x)R(x)$?

Exercise 8. HMMT. If $f(x)$ is monic quartic polynomial such that $f(-1) = -1$, $f(2) = -4$, $f(-3) = -9$, and $f(4) = -16$, find $f(1)$.

Exercise 9. Mandelbrot. The equation $x^3 - 4x^2 + 5x - 1.9 = 0$ has real roots r , s , and t . Find the length of an interior diagonal of a box with sides r , s , and t .

Exercise 10. HMMT 2004. Let x be a real number such that $x^3 + 4x = 8$. Determine the value of $x^7 + 64x^2$.

Challenge Problems

Exercise 11. Three pirates found a treasure. Each has his own opinion of the value of pieces. How can they divide the treasure so that each believes that he got at least one third? Solve the same problem for N pirates.

Exercise 12. HMMT 2004. Find all real solutions to $x^4 + (2 - x)^4 = 34$.

Exercise 13. Below are some Turkish words, followed by their English translations. (The letter ç represents a sound similar to ch in *chop*.) kilimler — carpets, kilimde — in the carpet, deftere — to the notebook, defterlerde — in the notebooks, adamlara — to the men, taraffarda — in the sides, okula — to the school, arabalar — cars, deniz — ocean, masa — table, ev — house, havuç — carrot, defter — notebook. Give the Turkish translations for the following: in the ocean, tables, to the houses, to the carrot.