

Translations

Tanya Khovanova

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Class Discussion

M.C. Escher. Translations. Logic Review.

Given a vector a , the slide of a figure in the direction of a at a distance equal to the length of a is called a *translation*.

Theorem. A translation F' of a figure F is congruent to F .

Warm Up

Exercise 1. Three horses were galloping at 27 miles per hour. What was the speed of one horse?

Exercise 2. Every point on the plane is either black or white. Prove that there are two points of the same color at a distance of one meter.

Problem Set

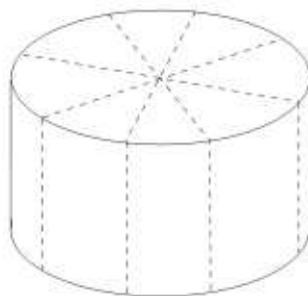
Exercise 3. There are n points on the plane. Prove that the shortest zig-zag line connecting all of them doesn't intersect itself.

Exercise 4. Prove that a translation of a circle is a circle.

Exercise 5. Given two circles R and S and a segment AB , build a segment that is equal and parallel to AB with the end points belonging to the given circles.

Exercise 6. Find the locus of points M the sum of whose distances from two given lines l_1 and l_2 is equal to a given value a .

Exercise 7. It's easy to see that a cylinder of cheese can be cut into eight pieces with four straight cuts. What is the maximum number of pieces you



can get with three straight cuts?

Exercise 8. Can a cube of cheese three inches on a side be cut into 27 one-inch cubes with five straight cuts? What if one can move the pieces prior to cutting?

Exercise 9. In this problem, “truthtellers” always tell the truth and “liars” always lie. In (1)-(3), all persons are either truthtellers or liars.

1. There are two persons, A and B. A says, “At least one of us is a liar.” What are A and B?
2. A says, “Either I am a liar or B is a truthteller.” What are A and B?
3. Now we have three persons, A, B, and C. A says, “All of us are liars.” B says, “Exactly one of us is a truthteller.” What are A, B, and C?
4. Now we have a third type of person, called “normal,” who sometimes lies and sometimes tells the truth. A says, “I am normal.” B says, “That is true.” C says, “I am not normal.” Exactly one of A, B, C is a truthteller, one is a liar, and one is normal. What are A, B, and C?